1. **True/False**: Circle the correct answer. No justifications are needed in this exercise.

(1) Every bounded sequence \((a_n)\) is Cauchy. \(\text{T} / \text{F}\)

(2) Assume that \((a_n)\) is a bounded sequence of real numbers which does not converge. Then \(\lim \inf a_n < \lim \sup a_n\). \(\text{T} / \text{F}\)

(3) Every sequence \((a_n)\) has a convergent subsequence. \(\text{T} / \text{F}\)

(4) Every bounded sequence \((a_n)\) has a Cauchy subsequence. \(\text{T} / \text{F}\)

(5) The series \(\sum_{n=1}^{\infty} \frac{n!}{8^n}\) is convergent. \(\text{T} / \text{F}\)

2. (a) Define what it means for a sequence \((a_n)\) of real numbers to be Cauchy.
(b) Let \((a_n)\) be a sequence of real numbers. Define what it means for a real number \(a\) to be a subsequential limit of \((a_n)\).
(c) Let \((a_n)_{n=1}^{\infty}\) be a sequence of real numbers. Define what it means for the series \(\sum_{n=1}^{\infty} a_n\) to be absolutely convergent.

3. (a) Let \((a_n)\) and \((b_n)\) be sequences of real numbers such that \(|b_m - b_n| \leq |a_m - a_n|\), for every \(m, n \in \mathbb{N}\). Assume that \((a_n)\) is convergent. Prove that \((b_n)\) is convergent.
(b) Let \((a_n)\) be the sequence given by \(a_n = \frac{(-1)^n n}{2n + 1}\). Prove that \(\lim \sup a_n = \frac{1}{2}\).

4. (a) Let \((a_n)\) be an unbounded sequence of real numbers. Prove that there exists a subsequence \((a_{n_k})\) of \((a_n)\) such that \(\lim_{k \to \infty} |a_{n_k}| = +\infty\).
(b) Let \((a_n)\) be a sequence of real numbers which does not converge to a real number \(a\). Prove that there exist \(\varepsilon > 0\) and a subsequence \((a_{n_k})\) of \((a_n)\) such that \(|a_{n_k} - a| \geq \varepsilon\), for every \(k \in \mathbb{N}\).

5. (a) Given an example of a convergent but not absolutely convergent series \(\sum_{n=1}^{\infty} a_n\).
(b) Prove that if \(\sum_{n=1}^{\infty} a_n\) is a convergent series of positive numbers, then the series \(\sum_{n=1}^{\infty} a_n^2\) converges.
(c) Let \((a_n)\) be a sequence of real numbers. Assume that the series \(\sum_{n=1}^{\infty} a_n b_n\) converges, for any bounded sequence \((b_n)\). Prove that the series \(\sum_{n=1}^{\infty} a_n\) is absolutely convergent.