Name: ______________________________________
PID: ____________________________________
Discussion Section: ___________________

Math 109 : Midterm 1
Introduction to Mathematical Reasoning

You have 1 hour and 20 minutes. You may not use any electronic devices, books, notes.

For every exercise you are given two pages, the second of which you may use for rough work.

1. (20 points)

(a) (10 points) Use a truth table to show that, for all statements P, Q and R, the statement '(P and Q) $\Rightarrow$ R' is equivalent to the statement '(P $\Rightarrow$ R) or (Q $\Rightarrow$ R)'.

(b) (10 points) Consider the following statement about the polynomial $f(x) = x^4 + x + 1$: 'for all real numbers x we have $f(x) \geq 0$'. Write down the negation of this statement.

2. (15 points) Prove that if x is a positive real number with $x \neq 1$, then $x + \dfrac{1}{x} > 2$. For each property you use, you must name it or write out a statement of it.

3. (15 points) Prove that there do not exist integers m and n such that $6m + 15n = 2$.

4. (15 points) Prove that for all positive integers n we have

$$\sum_{i=0}^{n}(2i+1) = (n+1)^2.$$

5. (20 points) For a positive integer n the number a_n is defined inductively by

$$a_1 = \frac{1}{2},$$

$$a_2 = 1,$$

$$a_{k+1} = \frac{a_k^2 + a_{k-1} + 4}{5} \quad \text{for any integer } k \geq 2.$$

(a) (10 points) Prove that $a_n < 2$ for all positive integers n.

(b) (10 points) Prove that $a_n < a_{n+1}$ for all positive integers n.

You may assume the following fact: if x, y are positive real numbers with $x < y$, then $x^2 < y^2$.

6. (15 points) Let A, B and C be subsets of some universal set U. Prove that

$$A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C).$$

Do not use truth tables.

Do not write on this page.

1		out of 20 points
2		out of 15 points
3		out of 15 points
4		out of 15 points
5		out of 20 points
6		out of 15 points
Total		out of 100 points