Show all of your work. Full credit will be given only for answers with explanations.

1. (100 points) Check all the correct statements.
   
   - $\mathbf{u} \cdot \mathbf{v} = -7$, where $\mathbf{u} = \langle 1, 2, 7 \rangle$ and $\mathbf{v} = \langle 4, -2, -1 \rangle$.
   - Length of the projection of the vector $\langle 2, 2, 7 \rangle$ on the line going through the vector $\langle 3, 6, 2 \rangle$ is equal to $\frac{32}{7}$.
   - The angle between the vector $\langle 1, 1, 1 \rangle$ and $\langle 1, 1, 0 \rangle$ is equal to $\arccos \frac{2}{\sqrt{3}}$.
   - $\mathbf{u} \times \mathbf{v} = \mathbf{w}$, where $\mathbf{u} = \langle 1, 1, 0 \rangle$, $\mathbf{v} = \langle 1, 2, 0 \rangle$ and $\mathbf{w} = \langle 1, -1, 0 \rangle$.
   - The vector $\langle 1, 3, 5 \rangle$ is the direction of the line defined by the equation
     \[ \frac{x - 1}{2} = \frac{y - 3}{3} = \frac{z - 5}{4}. \]

Solution:

1. $\mathbf{u} \cdot \mathbf{v} = 1 \cdot 4 + 2 \cdot (-2) + 7 \cdot (-1) = 4 - 4 - 7 = 7$. Hence, the statement is true.

2. Length of the projection of the vector $\mathbf{u} = \langle 2, 2, 7 \rangle$ on the line going through the vector $\mathbf{v} = \langle 3, 6, 2 \rangle$ is equal to $\frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}} = \frac{2 + 12 + 14}{\sqrt{9 + 36 + 4}} = \frac{32}{7}$. Hence, the statement is not true.

3. Let $\mathbf{u} = \langle 1, 1, 1 \rangle$ and $\mathbf{v} = \langle 1, 1, 0 \rangle$. Note that the angle between these two vectors is equal to $\arccos \frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{u} \cdot \mathbf{u}} = \frac{2}{\sqrt{3}}$. Hence, the statement is true.

4. $\mathbf{u} \times \mathbf{v} = (1 \cdot 0 - 2 \cdot 0)i - (1 \cdot 0 - 0 \cdot 0)j + (1 \cdot 1 - 1 \cdot 1)k = k$.

5. The statement is not true, since the denominators should be equal to the components of the direction of the line.
2. Let \( A = \langle 2, 0, 0 \rangle, B = \langle 0, 4, 0 \rangle \).

(a) (10 points) Find a direction vector of the line that goes through the points \( A \) and \( B \).

\[
\text{Solution: } \text{Note that the line goes in the direction } \vec{AB} = \langle -2, 4, 0 \rangle.
\]

(b) (10 points) Find a parametric form of the line that goes through the points \( A \) and \( B \).

\[
\text{Solution: } \text{The parametric form of a line is } \mathbf{r} = \mathbf{r}_0 + tv \text{ where } \mathbf{v} \text{ is the direction and } \mathbf{r}_0 \text{ is some point from the line. Hence, the parametric form of the line that goes throw the points } A \text{ and } B \text{ is } \mathbf{r} = \langle -2t, 4 + 4t, 0 \rangle.
\]

(c) (10 points) Find an equation of the line that goes through the points \( A \) and \( B \).

\[
\text{Solution: } \text{The equation of the line is } \begin{cases} -\frac{x}{2} = \frac{y-4}{4} \\ z = 0 \end{cases} \text{ since the parametric form of the line is } \langle x, y, z \rangle = \langle -2t, 4 + 4t, 0 \rangle.
\]
3. (10 points) Find \( u \times v \), where \( u = \langle 1, 1, 0 \rangle \), \( v = \langle 1, 0, 1 \rangle \)

Solution: Note that \( u = i + j \) and \( v = i + k \). Hence, \( u \times v = i \times k + j \times i + j \times k = -j - k + i = \langle 1, -1, -1 \rangle \).
4. Let $A = \langle 1, -1, 2 \rangle$, $B = \langle -1, 0, 1 \rangle$, and $C = \langle 0, 2, 1 \rangle$.

(a) (10 points) Find a vector $n$ which is perpendicular to the plane that goes through the points $A$, $B$, and $C$.

Solution: Let $u = \vec{AB} = \langle -1 - 1, 0 + 1, 1 - 2 \rangle = \langle -2, 1, -1 \rangle$ and $v = \vec{AC} = \langle 0 - 1, 2 + 1, 1 - 2 \rangle = \langle -1, 3, -1 \rangle$. Note that we just need to find a vector $n$ that is perpendicular to both $u$ and $v$. Recall that $u \times v$ is perpendicular to both $u$ and $v$. Hence, we may just choose $n = u \times v$. Hence, the result is $\langle -2, 1, -1 \rangle \times \langle -1, 3, -1 \rangle = (1 \cdot (-1) - 3 \cdot (-1))i - ((-2) \cdot (-1) - (-1) \cdot (-1))j + ((-2) \cdot 3 - 1 \cdot (-1))k = 2i - j - 5k = \langle 2, -1, -5 \rangle$.

(b) (10 points) Find the equation of the plane passing through the points $A$, $B$, and $C$.

Solution: Note that a vector $v = \langle x, y, z \rangle$ is perpendicular to $n$ iff $v \cdot n = 0$. In other words a point $P$ belongs to the plane iff $\vec{AP} \cdot n = 0$. As a result, the equation of the plane is $2(x - 1) - (y + 1) - 5(z - 2) = 0.$