Show all of your work. Full credit will be given only for answers with explanations.

1. (100 points) Check all the correct statements.
   - $u \cdot v = -7$, where $u = (1, 2, 7)$ and $v = (4, -2, -1)$.
   - Length of the projection of the vector $(2, 2, 7)$ on the line going through the vector $(3, 6, 2)$ is equal to $\frac{32}{7}$.
   - The angle between the vector $(1, 1, 1)$ and $(1, 1, 0)$ is equal to $\arccos \frac{2}{\sqrt{6}}$.
   - $u \times v = w$, where $u = (1, 1, 0)$, $v = (1, 2, 0)$ and $w = (1, -1, 0)$.
   - The vector $(1, 3, 5)$ is the direction of the line defined by the equation
     
     \[
     \frac{x - 1}{2} = \frac{y - 3}{3} = \frac{z - 5}{4}.
     \]

Solution:

1. $u \cdot v = 1 \cdot 4 + 2 \cdot (-2) + 7 \cdot (-1) = 4 - 4 - 7 = -7$. Hence, the statement is true.

2. Length of the projection of the vector $u = (2, 2, 7)$ on the line going through the vector $v = (3, 6, 2)$ is equal to $\frac{u \cdot v}{|v|} = \frac{2 \cdot 3 + 2 \cdot 6 + 7 \cdot 2}{\sqrt{3^2 + 6^2 + 2^2}} = \frac{32}{7}$. Hence, the statement is not true.

3. Let $u = (1, 1, 1)$ and $v = (1, 1, 0)$. Note that the angle between these two vectors is equal to $\arccos \frac{u \cdot v}{|u||v|} = \frac{2}{\sqrt{2}}$. Hence, the statement is true.

4. $u \times v = (1 \cdot 0 - 2 \cdot 0)i - (1 \cdot 0 - 0 \cdot 0)j + (1 \cdot 2 - 1 \cdot 1)k = k$.

5. The statement is not true, since the denominators should be equal to the components of the direction of the line.
2. Let $A = (2, 0, 0), B = (0, 4, 0)$.

(a) (10 points) Find a direction vector of the line that goes through the points $A$ and $B$.

**Solution:** Note that the line goes in the direction $\vec{AB} = (-2, 4, 0)$.

(b) (10 points) Find a parametric form of the line that goes through the points $A$ and $B$.

**Solution:** The parametric form of a line is $r = r_0 + tv$ where $v$ is the direction and $r_0$ is some point from the line. Hence, the parametric form of the line that goes through the points $A$ and $B$ is $r = (-2t, 4 + 4t, 0)$.

(c) (10 points) Find an equation of the line that goes through the points $A$ and $B$.

**Solution:** The equation of the line is $\begin{cases} -\frac{x}{2} = \frac{y-4}{4} \\ z = 0 \end{cases}$ since the parametric form of the line is $(x, y, z) = (-2t, 4 + 4t, 0)$. 
3. (10 points) Find $u \times v$, where $u = \langle 1, 1, 0 \rangle$, $v = \langle 1, 0, 1 \rangle$

**Solution:** Note that $u = i + j$ and $v = i + k$. Hence, $u \times v = i \times k + j \times i + j \times k = -j - k + i = \langle 1, -1, -1 \rangle$. 
4. Let $A = \langle 1, -1, 2 \rangle$, $B = \langle -1, 0, 1 \rangle$, and $C = \langle 0, 2, 1 \rangle$.

(a) (10 points) Find a vector $n$ which is perpendicular to the plane that goes through the points $A$, $B$, and $C$.

**Solution:** Let $u = \vec{AB} = \langle -1 - 1, 0 + 1, 1 - 2 \rangle = \langle -2, 1, -1 \rangle$ and $v = \vec{AC} = \langle 0 - 1, 2 + 1, 1 - 2 \rangle = \langle -1, 3, -1 \rangle$. Note that we just need to find a vector $n$ that is perpendicular to both $u$ and $v$. Recall that $u \times v$ is perpendicular to both $u$ and $v$. Hence, we may just choose $n = u \times v$. The result is $\langle -2, 1, -1 \rangle \times \langle -1, 3, -1 \rangle = \langle 1 \cdot (-1) - 3 \cdot (-1), (-2) \cdot (-1) - (-1) \cdot (-1), (-2) \cdot 3 - 1 \cdot (-1) \rangle = \langle 2, -1, -5 \rangle$.

(b) (10 points) Find the equation of the plane passing through the points $A$, $B$, and $C$.

**Solution:** Note that a vector $v = \langle x, y, z \rangle$ is perpendicular to $n$ iff $v \cdot n = 0$. In other words a point $P$ belongs to the plane iff $\vec{AP} \cdot n = 0$. As a result, the equation of the plane is $2(x - 1) - (y + 1) - 5(z - 2) = 0$. 