Note that every statement in the homework should be proved. The only exceptions are statements that were proven in previous homework or midterms and statements proven earlier in the class.

1. Alice and Bob play the following game.
   - Initially, there are 20 numbers: 10 numbers 1 and 10 numbers 2.
   - On each step one of the players select two numbers; and if they were the same, replace them by 2; otherwise, replace them by 1.
   - Alice make the first move and they do moves one after another.

Alice wins if the last number is 1 and Bob wins if the last number is 2. Who is the winner?

Note that this game is not a combinatorial game.
2. In the subtraction game where players may subtract 1, 2 or 5 chips on their turn, identify the N and P positions.
3. Is the Nim position \((1, 3, 5)\) an N-position (explain your answer)?
4. Consider the Misère subtraction game where players may subtract 1, 5 or 6 chips on their turn, identify the N and P positions.