1. (10 points) Let \( a_n \) be a sequence such that \( a_1 = 9, a_2 = 41, \) and \( a_{n+2} = 9a_{n+1} - 20a_n. \) Show that \( a_n = 4^n + 5^n. \)
2. We say that $L$ is a list of powers of $x$ iff

- either $L = x^k$ for some positive integer $k$ or
- $L = (x^k, L')$ where $L'$ is a list of powers of $x$ and $k$ is a positive integer.

Let $L$ be a list of powers of $x$. We say that the sum of $L$ with $x = v$ denoted by $\sum L|_{x=v}$

- is equal to $v^k$ whether $L = x^k$ and
- is equal to $v^k + \sum L'|_{x=v}$ whether $L = (x^k, L')$.

Prove that for any list $L$ of powers of $x$ there is a polynomial such that $\sum L|_{x=v} = p(v)$ for all real numbers $v$. 

3. (10 points) Prove that \( \sum_{i=1}^{n} (i+1)2^i = n2^{n+1} \) for all integers \( n \geq 1 \).