1. (10 points) Let us define a union of more than two sets as follows. Let $A_1, \ldots, A_n$ be some sets. Then

- $\bigcup_{i=1}^{1} A_i = A_1$ and
- $\bigcup_{i=1}^{k+1} A_i = \left( \bigcup_{i=1}^{k} A_i \right) \cup A_{k+1}$.

Show that $\bigcup_{i=1}^{n} [i] = [n]$ for all integers $n > 0$. 

2. (10 points) Let us define an intersection of more than two sets as follows. Let $A_1, \ldots, A_n$ be some sets. Then
\[
\begin{align*}
\textbullet & \quad \bigcap_{i=1}^1 A_i = A_1 \\
\textbullet & \quad \bigcap_{i=1}^{k+1} A_i = \left( \bigcap_{i=1}^k A_i \right) \cap A_{k+1}.
\end{align*}
\]
Show that $\bigcap_{i=1}^n \{ x \in \mathbb{N} : i \leq x \leq n \} = \{ n \}$ for all integers $n > 0$. 