1. (10 points) Let $S = \{n \in \mathbb{N} : x^n + y^n = z^n \text{ has an integer solution}\}$. Prove that the set $S$ is enumerable.
2. (10 points) Let $A, B \subseteq \mathbb{N}$ be enumeratable. Show that $A \cup B$ is also enumeratable.
3. (10 points) We say that a real number $\alpha$ be computable iff there is a computable function $a : \mathbb{Q} \to \mathbb{Q}$ such that $|\alpha - a(\epsilon)| \leq \epsilon$ for any rational $\epsilon > 0$.

Show that a number $\alpha < 1$ is computable iff the function $f : \mathbb{N} \to \{0, 1, \ldots, 9\}$ such that $f(i)$ is the $i$th digit of the base-10 representation of $\alpha$ is computable.
4. (10 points) Let $X, A, B \subseteq \mathbb{N}$ such that $X = A \Delta B$ ($X$ is the symmetric difference of $A$ and $B$) and $A$ and $B$ are enumerable. Prove that there are $A', B' \subseteq \mathbb{N}$ such that $X = A' \setminus B'$ and $A'$ and $B'$ are enumerable.