1. (10 points) Check all the correct statements.

- The inverse of the permutation (1, 2)(2, 4, 5)(2, 3) is 51234.
- There are 27 different strings of length 3 over the alphabet with 3 letters.
- Product of the permutations 13254 and 12354 is 23154.
- There are 60 different surjective functions from [5] to [4].
- There are 3 ways to put 4 identical balls into 3 different boxes such that all the boxes are not empty.
- A graph on 4 vertices has at most 6 edges.
- A connected graph on 5 vertices has at most 5 edges.
- If a graph on 5 vertices has 3 edges it should be disconnected.
- The following graph has an Eulerian path.

```
A ——— B ——— C ——— E
    
D ——— F
```

- The following graph has a Hamiltonian cycle.

```
A ——— B ——— C ——— E
    
D ——— F
```
2. (10 points) What is the maximal number of edges of a simple graph $G$ on $[n]$ if it is not connected?
3. (10 points) Show that for any $n \in \mathbb{N}$, there are two permutations $p, q \in S_n$ such that any permutation from $S_n$ can be expressed as their product (we can use each permutation multiple times).
4. (10 points) Let \( a_n = 2a_{n-1} - a_{n-2} \) for \( n \geq 2 \), \( a_1 = 2 \), and \( a_0 = 1 \). Find a closed formula (no summation signs) for the recurrent sequence \( a_n \).
5. (10 points) Show that among any $n + 1$ positive integers not exceeding $2n$ there must be an integer that divides one of the other integers.
6. (10 points) Give a simple closed form expression for the sum

\[ \sum_{a+b+c=7 \atop a,b,c \geq 0} \binom{7}{a,b,c}. \]