1. (60 points) Check all the correct statements.
   □ The number of different strings you can get by reordering letters in the word aabbc is 30.
   □ There are 25 different strings of length 5 over the alphabet with two letters.
   □ If you have 26 balls in 5 boxes, then there is a box with at least 6 balls.
   □ There are 6 different surjective functions from [3] to [3].
   □ The following graph is connected.

   ![Diagram of a graph with vertices A, B, C, D and edges connecting A to B, B to C, and D to A]

   □ A graph on 4 vertices has at most 6 edges.
2. In this exercise $F(x, y)$ is a predicate “x is shorter than y”. Write each proposition in words.

(a) (10 points) $\exists x \forall y F(x, y)$

(b) (10 points) $\forall x \forall y F(x, y)$
3. (10 points) Let us consider five-point geometry, it is a theory with undefined terms: point, line, is on, and axioms:

1. there exist exactly five points,
2. each two distinct points have exactly one line on both of them, and
3. each line has exactly two points.

Show that each point has exactly four lines on it.
4. (10 points) Prove the following recurrent formula:

\[ S(n, k) = S(n - 1, k - 1) + k \cdot S(n - 1, k), \]

where \( S(n, k) \) denotes the number of surjective functions from \([n]\) to \([k]\).
5. (10 points) Show that the following equality is always true:

\[
\binom{n + m}{k} = \sum_{i=0}^{k} \binom{n}{i} \cdot \binom{m}{k - i}.
\]
6. Let $a_1, \ldots, a_n$ be some integers.

(a) (10 points) Let us assume that for every $k \in [n]$, $\sum_{i=0}^{k} a_i \not\equiv 0 \pmod{n}$. Show that there are $k_1 \neq k_2$ such that $\sum_{i=0}^{k_1} a_i \equiv \sum_{i=0}^{k_2} a_i \pmod{n}$.

(b) (10 points) Prove that there are $k > 0$ and $\ell \geq 0$ such that $k + \ell \leq n$ and $\sum_{i=0}^{\ell} a_{k+i}$ is divisible by $n$. 