1. (a) (40 points) Let $X = 2^{\{1,\ldots,5\}}$, $Y = \{3,4\}$, and $R$ be a relation on $X$ such that

$$aRb \iff a \cup Y = b \cup Y.$$ 

Is $R$ an equivalence relation on $X$?

(b) (10 points) Let $X = \mathbb{R}$ and $R$ be a relation on $X$ such that

$$xRy \iff x - y \in \mathbb{Z}.$$ 

Is $R$ an equivalence relation on $X$?
2. (50 points) Let $X = \mathbb{Z} \times \mathbb{R}$ and $R$ be a relation on $X$ such that

$$(x_1, y_1)R(x_2, y_2) \iff (x_1 \leq x_2 \lor (x_1 = x_2 \land y_1 \leq y_2)).$$

Is $R$ a partial order on $X$?
3. (50 points) Let $p$ be a polynomial of even degree. Is it possible that $p$ is a bijection from $\mathbb{R}$ to $\mathbb{R}$?
4. (a) (40 points) Assume that a person invests 2000$ at 14 percent interest rate compounded annually. Let $A_n$ represent the amount at the end of $n$ years. Find a recurrence relation for $A_n$.

(b) (10 points) Let $S_n$ denotes the number of $n$-bit strings that does not contain 00 as a substring. Find a recurrence relation for $S_n$. 