Grading Policy

All this information is available on the website: math.ucsd.edu/~aknop/ucsd/18/2017/

- Homework 5 to 10% of the course grade.
Grading Policy

All this information is available on the website:
math.ucsd.edu/~aknop/ucsd/18/2017/

- Homework 5 to 10% of the course grade.
- Quizzes 5 to 10% of the course grade.
Grading Policy

All this information is available on the website: math.ucsd.edu/~aknop/ucsd/18/2017/

- Homework 5 to 10% of the course grade.
- Quizzes 5 to 10% of the course grade.
- MatLab 5 to 10% of the course grade.
Grading Policy

All this information is available on the website:
math.ucsd.edu/~aknop/ucsd/18/2017/

- Homework 5 to 10% of the course grade.
- Quizzes 5 to 10% of the course grade.
- MatLab 5 to 10% of the course grade.
- Two midterm exams 20% of the course grade each.
Grading Policy

All this information is available on the website:
math.ucsd.edu/~aknop/ucsd/18/2017/

- Homework 5 to 10% of the course grade.
- Quizzes 5 to 10% of the course grade.
- MatLab 5 to 10% of the course grade.
- Two midterm exams 20% of the course grade each.
- The final exam 35% of the course grade.
Homework

All this information is available on the website:
math.ucsd.edu/~aknop/ucsd/18/2017/

Every Friday (except the first one) you will have a hometask that cover material from previous Friday, Monday, and Wednesday.
Homework

All this information is available on the website: math.ucsd.edu/~aknop/ucsd/18/2017/

Every Friday (except the first one) you will have a hometask that cover material from previous Friday, Monday, and Wednesday.

We will use mymathlab.com for them.
Homework

All this information is available on the website: math.ucsd.edu/~aknop/ucsd/18/2017/

Every Friday (except the first one) you will have a hometask that cover material from previous Friday, Monday, and Wednesday.

We will use mymathlab.com for them. You will need an access code for this.
Quizzes

All this information is available on the website:
math.ucsd.edu/˜aknop/ucsd/18/2017/

Every Friday (except the first one) you will have a quiz that cover material from previous Friday, Monday, and Wednesday.
Quizzes

All this information is available on the website: math.ucsd.edu/~aknop/ucsd/18/2017/

Every Friday (except the first one) you will have a quiz that cover material from previous Friday, Monday, and Wednesday. Quiz will take place on the last ten minutes of the class.
Quizzes

All this information is available on the website: math.ucsd.edu/~aknop/ucsd/18/2017/

Every Friday (except the first one) you will have a quiz that cover material from previous Friday, Monday, and Wednesday. Quiz will take place on the last ten minutes of the class. For grading these quizzed I will use Gradescope.
Discussions and Questions

All this information is available on the website: math.ucsd.edu/~aknop/ucsd/18/2017/

▶ email: aknop@ucsd.edu
Discussions and Questions

All this information is available on the website:
math.ucsd.edu/~aknop/ucsd/18/2017/

▶ email: aknop@ucsd.edu
▶ Piazza site:
  https://piazza.com/uc_san_diego/fall2017/math18/home
A linear equation in the variables $x_1, \ldots, x_n$ is an equation that can be written in the form

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b$$

where $a_1, \ldots, a_n,$ and $b$ are real numbers (i.e. $a_1, \ldots, a_n, b \in \mathbb{R}$).
Linear Equations

DEFINITION

A linear equation in the variables $x_1, \ldots, x_n$ is an equation that can be written in the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

where $a_1, \ldots, a_n,$ and $b$ are real numbers (i.e. $a_1, \ldots, a_n, b \in \mathbb{R}$).

EXAMPLE

1. Is $2x_1 + 4x_3 + x_5 = x_6$ a linear equation in the variables $x_1, \ldots, x_6$?
Linear Equations

**DEFINITION**

A *linear equation* in the variables $x_1, \ldots, x_n$ is an equation that can be written in the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

where $a_1, \ldots, a_n$, and $b$ are real numbers (i.e. $a_1, \ldots, a_n, b \in \mathbb{R}$).

**EXAMPLE**

1. $2x_1 + 4x_3 + x_5 = x_6$ is a linear equation in the variables $x_1, \ldots, x_6$ since it is equivalent to $2x_1 + 0x_2 + 4x_3 + x_5 + (-1)x_6 = 0$;
A linear equation in the variables $x_1, \ldots, x_n$ is an equation that can be written in the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

where $a_1, \ldots, a_n,$ and $b$ are real numbers (i.e. $a_1, \ldots, a_n, b \in \mathbb{R}$).

EXAMPLE

1. $2x_1 + 4x_3 + x_5 = x_6$ is a linear equation in the variables $x_1, \ldots, x_6$ since it is equivalent to $2x_1 + 0x_2 + 4x_3 + x_5 + (-1)x_6 = 0$;

2. Is $x_1 + \sqrt{5}x_2 = \sqrt{6}(1 - x_3)$ a linear equation in the variables $x_1, x_2, x_3$. 
Linear Equations

DEFINITION

A linear equation in the variables $x_1, \ldots, x_n$ is an equation that can be written in the form

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b$$

where $a_1, \ldots, a_n,$ and $b$ are real numbers (i.e. $a_1, \ldots, a_n, b \in \mathbb{R}$).

EXAMPLE

1. $2x_1 + 4x_3 + x_5 = x_6$ is a linear equation in the variables $x_1, \ldots, x_6$ since it is equivalent to $2x_1 + 0x_2 + 4x_3 + x_5 + (-1)x_6 = 0$;

2. $x_1 + \sqrt{5}x_2 = \sqrt{6}(1 - x_3)$ is a linear equation in variables $x_1, x_2, x_3$ since it is equivalent to $x_1 + \sqrt{5}x_2 + \sqrt{6}x_3 = \sqrt{6}$.
Linear Equations

DEFINITION

A **linear equation** in the variables $x_1, \ldots, x_n$ is an equation that can be written in the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

where $a_1, \ldots, a_n$, and $b$ are real numbers (i.e. $a_1, \ldots, a_n, b \in \mathbb{R}$).

EXAMPLE

1. Is $x_1 = x_2x_3 + x_4$ a linear equation in the variables $x_1, \ldots, x_4$?
Linear Equations

**DEFINITION**

A **linear equation** in the variables $x_1, \ldots, x_n$ is an equation that can be written in the form

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b$$

where $a_1, \ldots, a_n,$ and $b$ are real numbers (i.e. $a_1, \ldots, a_n, b \in \mathbb{R}$).

**EXAMPLE**

1. $x_1 = x_2 x_3 + x_4$ is not a linear equation in the variables $x_1, \ldots, x_4$ since we have a summand $x_2 x_3$;
Linear Equations

**DEFINITION**

A **linear equation** in the variables $x_1, \ldots, x_n$ is an equation that can be written in the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

where $a_1, \ldots, a_n,$ and $b$ are real numbers (i.e. $a_1, \ldots, a_n, b \in \mathbb{R}$).

**EXAMPLE**

1. $x_1 = x_2x_3 + x_4$ is not a linear equation in the variables $x_1, \ldots, x_4$ since we have a summand $x_2x_3$;
2. Is $x_1 = \sqrt{x_2}$ a linear equation in the variables $x_1$ and $x_2$?
Linear Equations

DEFINITION

A linear equation in the variables $x_1, \ldots, x_n$ is an equation that can be written in the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

where $a_1, \ldots, a_n$, and $b$ are real numbers (i.e. $a_1, \ldots, a_n, b \in \mathbb{R}$).

EXAMPLE

1. $x_1 = x_2x_3 + x_4$ is not a linear equation in the variables $x_1, \ldots, x_4$ since we have a summand $x_2x_3$;

2. $x_1 = \sqrt{x_2}$ is not a linear equation in the variables $x_1$ and $x_2$ because of presence of $\sqrt{x_2}$. 
A **linear equation** in the variables $x_1, \ldots, x_n$ is an equation that can be written in the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

where $a_1, \ldots, a_n$, and $b$ are real numbers (i.e. $a_1, \ldots, a_n, b \in \mathbb{R}$).

**EXAMPLE**

1. Is $x_1 = x_2x_3 + x_4$ a linear equation in the variables $x_1, x_2$?
Linear Equations

DEFINITION

A **linear equation** in the variables $x_1, \ldots, x_n$ is an equation that can be written in the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

where $a_1, \ldots, a_n,$ and $b$ are real numbers (i.e. $a_1, \ldots, a_n, b \in \mathbb{R}$).

EXAMPLE

1. $x_1 = x_2x_3 + x_4$ is a linear equation in the variables $x_1, x_2,$
Linear Equations

DEFINITION

A linear equation in the variables \( x_1, \ldots, x_n \) is an equation that can be written in the form

\[
a_1x_1 + a_2x_2 + \cdots + a_nx_n = b
\]

where \( a_1, \ldots, a_n \), and \( b \) are real numbers (i.e. \( a_1, \ldots, a_n, b \in \mathbb{R} \)).

EXAMPLE

1. \( x_1 = x_2x_3 + x_4 \) is a linear equation in the variables \( x_1, x_2 \);
2. Is \( x + y = 3 \) a linear equation in the variables \( x, y \)?
**Linear Equations**

**DEFINITION**

A *linear equation* in the variables $x_1, \ldots, x_n$ is an equation that can be written in the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

where $a_1, \ldots, a_n$, and $b$ are real numbers (i.e. $a_1, \ldots, a_n, b \in \mathbb{R}$).

**EXAMPLE**

1. $x_1 = x_2x_3 + x_4$ is a linear equation in the variables $x_1, x_2$;
2. $x + y = 3$ is a linear equation in the variables $x, y$. 
DEFINITION

A system of linear equations in the variables $x_1, \ldots, x_n$ is a collection of one or more linear equations in the variables $x_1, \ldots, x_n$. 
DEFINITION

A system of linear equations in the variables \( x_1, \ldots, x_n \) is a collection of one or more linear equations in the variables \( x_1, \ldots, x_n \).

EXAMPLE

\[
\begin{align*}
2x_1 - x_2 + 3x_3 &= 8 \\
x_1 - 4x_3 &= -7
\end{align*}
\]
Solutions of a System of Linear Equations

**DEFINITION**

A solution of the system of linear equations in the variables $x_1, \ldots, x_n$ is a list $(s_1, \ldots, s_n)$ of real numbers such that each equation in the system became true if we substitute the values $s_1, \ldots, s_n$ to the variables $x_1, \ldots, x_n$ respectively.

**EXAMPLE**

$(5; 6; 5; 3)$ is a solution of
\[
\begin{align*}
2x_1 + x_2 + 1 &= 5 \\
5x_3 &= 8 \\
4x_3 &= 7
\end{align*}
\]

since
\[
\begin{align*}
2 \cdot 5 + 6 + 1 &= 8 \\
5 \cdot 3 &= 15 \\
4 \cdot 3 &= 12
\end{align*}
\]
Solutions of a System of Linear Equations

**DEFINITION**

A solution of the system of linear equations in the variables $x_1, \ldots, x_n$ is a list $(s_1, \ldots, s_n)$ of real numbers such that each equation in the system became true if we substitute the values $s_1, \ldots, s_n$ to the variables $x_1, \ldots, x_n$ respectively.

**EXAMPLE**

$(5, 6.5, 3)$ is a solution of

\[
\begin{align*}
2x_1 - x_2 + 1.5x_3 &= 8 \\
x_1 - 4x_3 &= -7 
\end{align*}
\]

since $2 \cdot 5 - 6.5 + 1.5 \cdot 3 = 8$ and $5 - 4 \cdot 3 = -7$. 
DEFINITION

The set of all possible solutions of a system of linear equations is called a solution set.
Solutions of a System of Linear Equations

Solution set of the system

\[
\begin{align*}
    x_1 - 2x_2 &= -1 \\
    -x_1 + 3x_2 &= 3
\end{align*}
\]

is \{ (2, 3) \}. 

\[
\begin{array}{c|c}
0 & 1 \\
1 & 2 \\
2 & 3 \\
3 & 4 \\
4 & 5 \\
\end{array}
\]

\[
\begin{array}{c|c}
0 & 0 \\
1 & 1 \\
2 & 2 \\
3 & 3 \\
\end{array}
\]
Solutions of a System of Linear Equations

Solution set of the system

\[ x_1 - 2x_2 = -1 \]
\[ x_1 - 2x_2 = 0 \]

is empty.
Solutions of a System of Linear Equations

Solution set of the system

\[
\begin{align*}
    x_1 - 2x_2 &= -1 \\
    2x_1 - 4x_2 &= -2
\end{align*}
\]

is equal to \( \{( -1 + 2x_2, x_2) \mid x_2 \in \mathbb{R} \} \).
Solutions of a System of Linear Equations

A system of two linear equations in two variables has

1. no solution, or
2. exactly one solution, or
3. infinitely many solutions.
Solutions of a System of Linear Equations

A system of linear equations has

1. no solution, or
2. exactly one solution, or
3. infinitely many solutions.
Matrix notation

The most important information about a system of equations we can write in a rectangular array.

EXAMPLE

The system

\[
\begin{align*}
x_1 - 2x_2 + 4x_3 &= -1 \\
2x_1 + x_3 &= -2
\end{align*}
\]

We may rewrite as

\[
\begin{bmatrix}
1 & -2 & 4 & -1 \\
2 & 0 & 1 & -2
\end{bmatrix}
\]

We call such a matrix **augmented matrix** and this matrix without last column is called **coefficient matrix**.
Solving a Linear System

Let us solve the following system.

\[
\begin{align*}
    x_1 - 2x_2 + x_3 &= 0 \\
    2x_2 - 8x_3 &= 8 \\
    -4x_1 + 5x_2 + 9x_3 &= -9
\end{align*}
\]

\[
\begin{bmatrix}
    1 & -2 & 1 & 0 \\
    0 & 2 & -8 & 8 \\
    -4 & 5 & 9 & -9
\end{bmatrix}
\]
Solving a Linear System

\[\begin{align*}
x_1 - 2x_2 + x_3 &= 0 \\
2x_2 - 8x_3 &= 8 \\
-4x_1 + 5x_2 + 9x_3 &= -9
\end{align*}\]

Add 4 times the first equation to the third equation.

\[
\begin{align*}
4 \cdot [\text{equation 1:}] + [\text{equation 3:}] &= \\
4x_1 - 8x_2 + 4x_3 &= 0 \\
-4x_1 + 5x_2 + 9x_3 &= -9 \\
-3x_2 + 13x_3 &= -9
\end{align*}
\]
Solving a Linear System

\[
\begin{align*}
    x_1 - 2x_2 + x_3 &= 0 \\
    2x_2 - 8x_3 &= 8 \\
    -3x_2 + 13x_3 &= -9
\end{align*}
\]

\[
\begin{bmatrix}
    1 & -2 & 1 & 0 \\
    0 & 2 & -8 & 8 \\
    0 & -3 & 13 & -9
\end{bmatrix}
\]
Solving a Linear System

Let us solve the following system.

\[ x_1 - 2x_2 + x_3 = 0 \]
\[ 2x_2 - 8x_3 = 8 \]
\[ -3x_2 + 13x_3 = -9 \]

Divide equation 2 by 2.

\[ x_1 - 2x_2 + x_3 = 0 \]
\[ x_2 - 4x_3 = 4 \]
\[ -3x_2 + 13x_3 = -9 \]

\[
\begin{bmatrix}
1 & -2 & 1 & 0 \\
0 & 2 & -8 & 8 \\
0 & -3 & 13 & -9 \\
\end{bmatrix}
\]
Solving a Linear System

\begin{align*}
x_1 - 2x_2 + x_3 &= 0 \\
x_2 - 4x_3 &= 4 \\
-3x_2 + 13x_3 &= -9
\end{align*}

\[
\begin{bmatrix}
1 & -2 & 1 & 0 \\
0 & 1 & -4 & 4 \\
0 & -3 & 13 & -9
\end{bmatrix}
\]

Add 3 times equation 2 to equation 3.

\[
\begin{align*}
ex_2 &+ 12x_3 = 12 \\
x_3 &= 3
\end{align*}
\]
Solving a Linear System

Let us solve the following system.

\[
\begin{align*}
    x_1 - 2x_2 + x_3 &= 0 \\
    x_2 - 4x_3 &= 4 \\
   -3x_2 + 13x_3 &= -9
\end{align*}
\]

Add 3 times equation 2 to equation 3.

\[
\begin{align*}
    3 \cdot [\text{equation 2:}] + [\text{equation 3:}] &= \begin{align*}
        3x_2 - 12x_3 &= 12 \\
        -3x_2 + 13x_3 &= -9
    \end{align*}
\end{align*}
\]

[new equation 3:]

\[
x_3 = 3
\]
Solving a Linear System

\begin{align*}
    x_1 - 2x_2 + x_3 &= 0 \\
    x_2 - 4x_3 &= 4 \\
    x_3 &= 3
\end{align*}

\[
\begin{bmatrix}
    1 & -2 & 1 & 0 \\
    0 & 1 & -4 & 4 \\
    0 & 0 & 1 & 3
\end{bmatrix}
\]
Solving a Linear System

Let us solve the following system.

\[
\begin{align*}
    x_1 - 2x_2 + x_3 &= 0 \\
    x_2 - 4x_3 &= 4 \\
    x_3 &= 3
\end{align*}
\]

Add \(2\) times equation 2 to equation 1.

\[
\begin{align*}
    2 \cdot [\text{equation 2:}] & & 2x_2 - 8x_3 &= 8 \\
    + [\text{equation 1:}] & & x_1 - 2x_2 + x_3 &= 0 \\
    \text{[new equation 1:]} & & x_1 - 7x_3 &= 8
\end{align*}
\]
Let us solve the following system.

\[
\begin{align*}
    x_1 - 7x_3 &= 8 \\
    x_2 - 4x_3 &= 4 \\
    x_3 &= 3
\end{align*}
\]

\[
\begin{bmatrix}
    1 & 0 & -7 & 8 \\
    0 & 1 & -4 & 4 \\
    0 & 0 & 1 & 3
\end{bmatrix}
\]

Add 4 times equation 3 to equation 2.

\[
\begin{align*}
    4x_3 &= 12 \\
    x_2 &= 16
\end{align*}
\]
Solving a Linear System

Let us solve the following system.

\[
\begin{align*}
    x_1 & - 7x_3 = 8 \\
    x_2 & - 4x_3 = 4 \\
    x_3 & = 3
\end{align*}
\]

Add 4 times equation 3 to equation 2.

\[
\begin{align*}
    4 \cdot [\text{equation 3:}] & \quad 4x_3 = 12 \\
    + [\text{equation 1:}] & \quad x_2 - 3x_3 = 4 \\
    \text{[new equation 2:]} & \quad x_2 = 16
\end{align*}
\]
Solving a Linear System

\[\begin{align*}
x_1 - 7x_3 &= 8 \\
x_2 &= 16 \\
x_3 &= 3
\end{align*}\]

\[
\begin{bmatrix}
1 & 0 & -7 & 8 \\
0 & 1 & 0 & 16 \\
0 & 0 & 1 & 3
\end{bmatrix}
\]
Let us solve the following system.

\[
\begin{align*}
x_1 - 7x_3 &= 8 \\
x_2 &= 16 \\
x_3 &= 3
\end{align*}
\]

Add 7 times equation 3 to equation 1.

\[
\begin{align*}
7 \cdot [\text{equation 3:}] + [\text{equation 1:}] & \quad \Rightarrow \quad [\text{new equation 1:}] \\
7x_3 &= 21 \\
x_1 - 7x_3 &= 8 \\
x_1 &= 29
\end{align*}
\]
Solving a Linear System

Let us solve the following system.

\[ \begin{align*}
    x_1 & = 29 \\
    x_2 & = 16 \\
    x_3 & = 3
\end{align*} \]
General Case

ELEMENTARY ROW OPERATIONS

Replacement  Replace one row by the sum of itself and multiple of another row.
Interchange  Interchange two rows.
Scaling  Multiply a row by a nonzero constant.

Note that we may apply these operations to any matrix, not only to the augmented matrix of a system of equations.
General Case

ELEMENTARY ROW OPERATIONS

Replacement  Replace one row by the sum of itself and multiple of another row.

Interchange  Interchange two rows.

Scaling  Multiply a row by a nonzero constant.

Note that we may apply these operations to any matrix. Not only to the augmented matrix of a system of equations.
General Case

ELEMENTARY ROW OPERATIONS

Replacement  Replace one row by the sum of itself and multiple of another row.

Interchange  Interchange two rows.

Scaling  Multiply a row by a nonzero constant.

Note that we may apply these operations to any matrix. Not only to the augmented matrix of a system of equations.

DEFINITION

We call two matrices row equivalent if there is a sequence of elementary row operations that transform one matrix into the other.
Conservation of a Solutions Set

THEOREM

If the augmented matrices of two systems are row equivalent, then two systems have the same solutions sets.
Conservation of a Solutions Set

THEOREM

If the augmented matrices of two systems are row equivalent, then two systems have the same solutions sets.

TWO FUNDAMENTAL QUESTIONS ABOUT A LINEAR SYSTEM

1. Is the system consistent; that is, does at least one solution exist?
2. If solution exists, is it the only one?; that is, is the solution unique?
The Fundamental Questions

**EXAMPLE**

Let us answer this question about the system

\[ x_2 - 4x_3 = 8 \]
\[ 2x_1 - 3x_2 + 2x_3 = 1 \]
\[ 5x_1 - 8x_2 + 7x_3 = 1 \]
EXAMPLE

Let us answer this question about the system

\[ x_2 - 4x_3 = 8 \]
\[ 2x_1 - 3x_2 + 2x_3 = 1 \]
\[ 5x_1 - 8x_2 + 7x_3 = 1 \]

Augmented matrix of this system is

\[
\begin{bmatrix}
0 & 1 & -4 & 8 \\
2 & -3 & 2 & 1 \\
5 & -8 & 7 & 1 \\
\end{bmatrix}
\]
The Fundamental Questions

EXAMPLE

Let us answer this question about the system

\[
\begin{bmatrix}
0 & 1 & -4 & 8 \\
2 & -3 & 2 & 1 \\
5 & -8 & 7 & 1
\end{bmatrix}
\]

Let us interchange rows 1 and 2.

\[
\begin{bmatrix}
2 & -3 & 2 & 1 \\
0 & 1 & -4 & 8 \\
5 & -8 & 7 & 1
\end{bmatrix}
\]
The Fundamental Questions

EXAMPLE

Let us answer this questions about the system

\[
\begin{bmatrix}
2 & -3 & 2 & 1 \\
0 & 1 & -4 & 8 \\
5 & -8 & 7 & 1 \\
\end{bmatrix}
\]

Add $-\frac{5}{2}$ times row 1 to row 3.

\[
\begin{bmatrix}
2 & -3 & 2 & 1 \\
0 & 1 & -4 & 8 \\
0 & -1/2 & 2 & -3/2 \\
\end{bmatrix}
\]
The Fundamental Questions

**EXAMPLE**

Let us answer this question about the system

\[
\begin{bmatrix}
2 & -3 & 2 & 1 \\
0 & 1 & -4 & 8 \\
0 & -1/2 & 2 & -3/2 \\
\end{bmatrix}
\]

Add 1/2 times row 2 to row 3.

\[
\begin{bmatrix}
2 & -3 & 2 & 1 \\
0 & 1 & -4 & 8 \\
0 & 0 & 0 & 5/2 \\
\end{bmatrix}
\]
The Fundamental Questions

EXAMPLE

Let us answer this questions about the system

\[
\begin{bmatrix}
2 & -3 & 2 & 1 \\
0 & 1 & -4 & 8 \\
0 & 0 & 0 & 5/2
\end{bmatrix}
\]

The corresponding system is

\[
\begin{align*}
2x_1 - 3x_2 + 2x_3 &= 1 \\
x_2 - 4x_3 &= 8 \\
+ 0 &= 5/2
\end{align*}
\]