Matrix operations

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Special Matrices

The scalar entry in the $i^{th}$ row and $j^{th}$ column of $A$ is denoted $a_{i,j}$ and is called the $(i,j)$-entry if $A$. 

- A diagonal matrix is a square $n \times n$ matrix whose nondiagonal entries are zero.

- A zero matrix is $m \times n$ matrix with all entries are equal to zero.
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- **diagonal entries** in a \(m \times n\) matrix \(A = [a_{i,j}]\) are \(a_{1,1}, a_{2,2}, \ldots\) and they form the **main diagonal** of \(A\).
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Sums

DEFINITION

If $A$ and $B$ are $m \times n$ matrices, then the sum $A + B$ is the $m \times n$ matrix whose columns are the sum of the corresponding columns of $A$ and $B$. 

EXAMPLE

Let $A = \begin{bmatrix} 4 & 0 & 5 \\ 1 & 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 7 \end{bmatrix}$. Then $A + B = \begin{bmatrix} 5 & 1 & 5 \\ 2 & 8 & 9 \end{bmatrix}$. 

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Let $A = \begin{bmatrix} 4 & 0 & 5 \\ -1 & 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 7 \end{bmatrix}$. Then $A + B = \begin{bmatrix} 5 & 1 & 5 \\ 2 & 8 & 9 \end{bmatrix}$. 
Scalar Multiples

DEFINITION

If $r$ is a scalar and $A$ is a matrix, then the scalar multiple $rA$ is the matrix whose columns are $r$ times the corresponding columns in $A$. 

EXAMPLE

Let $B = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 7 \end{bmatrix}$. Then $2B = \begin{bmatrix} 2 & 2 & 2 \\ 6 & 10 & 14 \end{bmatrix}$. 
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Matrix Multiplication
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\[
A (Bx) = Bx
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\[ x \rightarrow Bx \rightarrow A(Bx) \]
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Let $A$ be a $m \times n$ matrix, $B$ be a $n \times p$ matrix, and $x \in \mathbb{R}^p$. 

DEFINITION

If is a $m \times n$ matrix, $B$ be a $n \times p$ matrix, and $b_1, \ldots, b_p$ are columns of $B$, then the matrix $AB = [Ab_1; \ldots; Ab_p]$ is a matrix of size $m \times p$. 

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**EXAMPLE**

Let us compute $AB$ where $A = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 & 6 \\ 1 & -2 & 3 \end{bmatrix}$.
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$$Ab_1 = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 11 \\ -1 \end{bmatrix}, \quad Ab_2 = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 10 \\ 13 \end{bmatrix}, \quad Ab_3 = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \end{bmatrix} = \begin{bmatrix} 21 \\ -9 \end{bmatrix}.$$
Row-Column Rule

**THEOREM**

*If the product $AB$ is defined, then the entry in row $i$ and column $j$ of $AB$ is the sum of the products of corresponding entries from row $i$ of $A$ and column $j$ of $B$.***
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In other words if $(AB)_{i,j}$ denotes $(i,j)$-entry of $AB$ and $A$ is $m \times n$ matrix, then

$$a_{i,1}b_{1,j} + \cdots + a_{i,n}b_{n,j}.$$
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THEOREM

1. $A(BC) = (AB)C$
2. $A(B + C) = AB + AC$
3. $r(AB) = (rA)B = A(rB)$
Powers of Matrix

DEFINITION

If $A$ is $n \times n$ matrix, then $A^k$ denotes the product of $k$ copies of $A$. 
The Transpose of Matrix

**DEFINITION**

If $A$ is $m \times n$ matrix, then the **transpose** of $A$, denoted by $A^T$ is a matrix whose columns are formed from the corresponding rows of $A$. 
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**EXAMPLE**

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then $A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$
The Transpose of Matrix

**DEFINITION**

If $A$ is $m \times n$ matrix, then the **transpose** of $A$, denoted by $A^T$ is a matrix whose columns are formed from the corresponding rows of $A$.

**THEOREM**

Let $A$ and $B$ denote matrices whose size are appropriate for the following operations.

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$
3. for any scalar $r$, $(rA)^T = r(A^T)$