Catch up Review

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The Midterm Rules

- The midterm will last 50 minutes.
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Your seats are already assigned to you; you can find your seat on TritonEd.
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Preparations to the Midterm

- Make sure that you know how to solve every problem from your quizzes.
Preparations to the Midterm

- Make sure that you know how to solve every problem from your quizzes.
- Make sure that you understand statements of all theorems from the course.
Preparations to the Midterm

- Make sure that you know how to solve every problem from your quizzes.
- Make sure that you understand statements of all theorems from the course.
- Solve few problems from MyMathLab or from the book.
If I am not Prepared

Visit me or your TA during the office hours and ask questions!
Problems

PROBLEM

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & 2 \\ 2 & 4 & 3 \end{bmatrix}$ and $T(x) = Ax$. 

SOLUTION

Let us transform $A$ into reduced echelon form.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & 2 \\ 2 & 4 & 3 \end{bmatrix}$$

Transforming $A$ into reduced echelon form, we get:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore, $T(x) = Ax$ is an onto transformation.
Let \( A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & 2 \\ 2 & 4 & 3 \end{bmatrix} \) and \( T(x) = Ax \). Is \( T \) a onto transformation?
PROBLEM

Let \( A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & 2 \\ 2 & 4 & 3 \end{bmatrix} \) and \( T(x) = Ax \). Is \( T \) a onto transformation?

SOLUTION

\( T \) is a onto transformation iff columns of \( A \) span \( \mathbb{R}^3 \).
Problems

PROBLEM

Let \( A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & 2 \\ 2 & 4 & 3 \end{bmatrix} \) and \( T(x) = Ax \). Is \( T \) a onto transformation?

SOLUTION

\( T \) is a onto transformation iff columns of \( A \) span \( \mathbb{R}^3 \). In order to check this we have to check that \( Ax = b \) has a solution for every \( b \in \mathbb{R}^3 \).
Problems

PROBLEM

Let \( A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & 2 \\ 2 & 4 & 3 \end{bmatrix} \) and \( T(x) = Ax \). Is \( T \) a onto transformation?

SOLUTION

Let us transform \( \begin{bmatrix} 1 & 2 & 3 & b_1 \\ -1 & -1 & 2 & b_2 \\ 2 & 4 & 3 & b_3 \end{bmatrix} \) into reduced echelon form.

\[
\begin{bmatrix}
1 & 2 & 3 & b_1 \\
0 & 1 & 5 & b_2 + b_1 \\
0 & 0 & -3 & b_3 - 2b_1
\end{bmatrix}
\]
PROBLEM

Let \( A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & 2 \\ 2 & 4 & 3 \end{bmatrix} \) and \( T(x) = Ax \). Is \( T \) a onto transformation?

SOLUTION

Let us transform \( \begin{bmatrix} 1 & 2 & 3 & b_1 \\ -1 & -1 & 2 & b_2 \\ 2 & 4 & 3 & b_3 \end{bmatrix} \) into reduced echelon form.

\[
\begin{bmatrix} 1 & 2 & 3 & b_1 \\ 0 & 1 & 5 & b_2 + b_1 \\ 0 & 0 & -3 & b_3 - 2b_1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & b_1 \\ 0 & 1 & 5 & b_2 + b_1 \\ 0 & 0 & 1 & \frac{2b_1 - b_3}{3} \end{bmatrix}
\]
Let us consider the following system
\[ \begin{align*}
  x_1 + x_2 + x_3 &= 3 \\
  2x_1 - x_3 &= 4 \\
  -x_1 + 2x_2 - x_3 &= 6
\end{align*} \]
Find solutions in parametric vector form.

Hence, \( x_3 = \frac{4}{3} \), \( x_2 = 3 \), and \( x_1 = \frac{4}{3} \).

In parametric vector form solution is \( \begin{bmatrix} 2 \\ 4/3 \\ 3 \end{bmatrix} \).
PROBLEM

Let us consider the following system

\[
\begin{align*}
    x_1 + x_2 + x_3 &= 3 \\
    2x_1 - x_3 &= 4 \\
    -x_1 + 2x_2 - x_3 &= 6
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\]

Find solutions in parametric vector form.
Problems

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Let us consider the following system

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x_1 + x_2 + x_3 &= 3 \\
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\end{align*} \]

Find solutions in parametric vector form.

SOLUTION

Let us transform it into reduced echelon form.
Let us consider the following system

\[
\begin{align*}
&x_1 + x_2 + x_3 = 3 \\
&2x_1 - x_3 = 4 \\
&-x_1 + 2x_2 - x_3 = 6
\end{align*}
\]

Find solutions in parametric vector form.

Let us transform it into reduced echelon form.

\[
\begin{bmatrix}
1 & 1 & 1 & 3 \\
2 & 0 & -1 & 4 \\
-1 & 2 & -1 & 6
\end{bmatrix}
\]

Hence, \(x_3 = \frac{4}{3}\), \(x_2 = 3\), and \(x_1 = \frac{4}{3}\).

In parametric vector form solution is

\[
\begin{bmatrix}
\frac{4}{3} \\
3 \\
\frac{4}{3}
\end{bmatrix}
\]

Problems

PROBLEM

Let us consider the following system

\[ \begin{align*}
    x_1 + x_2 + x_3 &= 3 \\
    2x_1 - x_3 &= 4 \\
    -x_1 + 2x_2 - x_3 &= 6 
\end{align*} \]

Find solutions in parametric vector form.

SOLUTION

Let us transform it into reduced echelon form.

\[
\begin{bmatrix}
    1 & 1 & 1 & 3 \\
    2 & 0 & -1 & 4 \\
    -1 & 2 & -1 & 6 
\end{bmatrix}
\sim
\begin{bmatrix}
    1 & 1 & 1 & 3 \\
    0 & -2 & -3 & -2 \\
    0 & 3 & 0 & 9 
\end{bmatrix}
\]

Hence, 

\[ x_3 = \frac{4}{3}, \quad x_2 = 3, \quad \text{and} \quad x_1 = \frac{4}{3} \]

In parametric vector form solution is

\[ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{4}{3} \\ 3 \\ \frac{4}{3} \end{bmatrix} + \lambda \begin{bmatrix} 0 \\ -2 \\ -3 \end{bmatrix} + \mu \begin{bmatrix} 0 \\ -3 \\ 9 \end{bmatrix} \]

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Problems

PROBLEM

Let us consider the following system

\[
\begin{align*}
    x_1 + x_2 + x_3 &= 3 \\
    2x_1 - x_3 &= 4 \\
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\end{align*}
\]

Find solutions in parametric vector form.

SOLUTION

Let us transform it into reduced echelon form.

\[
\begin{bmatrix}
    1 & 1 & 1 & 3 \\
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    -1 & 2 & -1 & 6
\end{bmatrix}
\sim
\begin{bmatrix}
    1 & 1 & 1 & 3 \\
    0 & -2 & -3 & -2 \\
    0 & 3 & 0 & 9
\end{bmatrix}
\sim
\begin{bmatrix}
    1 & 1 & 1 & 3 \\
    0 & 1 & 0 & 3 \\
    0 & 0 & -3 & 4
\end{bmatrix}
\]

Hence,

\[
\begin{align*}
    x_3 &= 4/3 \\
    x_2 &= 3 \\
    x_1 &= 4/3
\end{align*}
\]

In parametric vector form solution is

\[
\begin{bmatrix}
    4/3 \\
    3 \\
    4/3
\end{bmatrix}
\]
Problem

**Problem**

Let us consider the following system

\[
\begin{align*}
    x_1 + x_2 + x_3 &= 3 \\
    2x_1 - x_3 &= 4 \\
    -x_1 + 2x_2 - x_3 &= 6
\end{align*}
\]

Find solutions in parametric vector form.

**Solution**

Hence, \( x_3 = -\frac{4}{3}, \ x_2 = 3, \) and \( x_1 = \frac{4}{3}. \) In parametric vector form solution is

\[
\begin{bmatrix}
    4/3 \\
    3 \\
    -4/3
\end{bmatrix}
\]
Let us consider the homogeneous version of the previous system

\begin{align*}
    x_1 + x_2 + x_3 &= 0 \\
    2x_1 - x_3 &= 0 \\
    -x_1 + 2x_2 - x_3 &= 0
\end{align*}

Find solutions in parametric vector form.

Any solution of this system plus some solution of the previous system is a solution of the previous system. Hence this system has only trivial solution, since there is only one solution of the previous system.
Problems

PROBLEM

Let us consider the homogeneous version of the previous system

\[ x_1 + x_2 + x_3 = 0 \]
\[ 2x_1 - x_3 = 0 \]
\[ -x_1 + 2x_2 - x_3 = 0 \]

Find solutions in parametric vector form.
Problems

PROBLEM

Let us consider the homogeneous version of the previous system
\[ x_1 + x_2 + x_3 = 0 \]
\[ 2x_1 - x_3 = 0 \]
\[ -x_1 + 2x_2 - x_3 = 0 \]

Find solutions in parametric vector form.

SOLUTION

Any solution of this system plus some solution of the previous system is a solution of the previous system.
Problems

PROBLEM

Let us consider the homogeneous version of the previous system
\[ x_1 + x_2 + x_3 = 0 \]
\[ 2x_1 - x_3 = 0 \]
\[ -x_1 + 2x_2 - x_3 = 0 \]
Find solutions in parametric vector form.

SOLUTION

Any solution of this system plus some solution of the previous system is a solution of the previous system. Hence this system has only trivial solution,
Problems

PROBLEM

Let us consider the homogeneous version of the previous system

\[ x_1 + x_2 + x_3 = 0 \]
\[ 2x_1 - x_3 = 0 \]
\[ -x_1 + 2x_2 - x_3 = 0 \]

Find solutions in parametric vector form.

SOLUTION

Any solution of this system plus some solution of the pervious system is a solution of the previous system. Hence this system has only trivial solution, since there is only one solution of the previous system.
Problems

PROBLEM

Let $T_1(x) = \begin{bmatrix} 1 & 1 & 1 \\ -5 & -4 & 1 \\ 4 & 3 & -2 \end{bmatrix} x$ and $T_2(x) = \begin{bmatrix} 1 & 3 & 2 \\ -2 & -5 & 4 \end{bmatrix} x$

Find the standard matrix for the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined as follows $T(x) = T_2(T_1(x))$. 
Problems

SOLUTION

The standard matrix of $T_1$ is \[
\begin{bmatrix}
1 & 1 & 1 \\
-5 & -4 & 1 \\
4 & 3 & -2
\end{bmatrix}
\] and standard matrix of $T_2$ is \[
\begin{bmatrix}
1 & 1 & 1 \\
-5 & -4 & 1 \\
4 & 3 & -2
\end{bmatrix}
\]

Since it is equivalent to \[
\begin{bmatrix}
1 & 1 & 1 \\
-5 & -4 & 1 \\
4 & 3 & -2
\end{bmatrix}
\]
Problems

**SOLUTION**

The standard matrix of $T_1$ is \[
\begin{bmatrix}
1 & 1 & 1 \\
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4 & 3 & -2
\end{bmatrix}
\] and standard matrix of $T_2$ is \[
\begin{bmatrix}
1 & 3 & 2 \\
-2 & -5 & 4
\end{bmatrix}.
\]

Hence the standard matrix of $T_2(T_1(x))$ is
\[
\begin{bmatrix}
1 & 3 & 2 \\
-2 & -5 & 4
\end{bmatrix} \begin{bmatrix}
1 & 1 & 1 \\
-5 & -4 & 1 \\
4 & 3 & -2
\end{bmatrix}
\]
Problems

**SOLUTION**

The standard matrix of $T_1$ is \[
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\end{bmatrix}.
\]

Hence, the standard matrix of $T_2(T_1(x))$ is

\[
\begin{bmatrix}
1 & 3 & 2 \\
-2 & -5 & 4
\end{bmatrix}
\begin{bmatrix}
1 & 1 & 1 \\
-5 & -4 & 1 \\
4 & 3 & -2
\end{bmatrix}
= \begin{bmatrix}
-6 & -5 & 0 \\
39 & 30 & -15
\end{bmatrix}
\]

Since it is equivalent to

\[
\begin{bmatrix}
1 & 3 & 2 \\
-2 & -5 & 4
\end{bmatrix}
\begin{bmatrix}
1 \\
-5 \\
4
\end{bmatrix}
\begin{bmatrix}
1 & 3 & 2 \\
-2 & -5 & 4 \\
3 & -2 & -5 & 4
\end{bmatrix}
\begin{bmatrix}
1 \\
-2
\end{bmatrix}.
\]
Problems

PROBLEM

Let \( S = \left\{ \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ k \end{bmatrix} \right\} \).
Problems

PROBLEM

Let \( S = \left\{ \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ k \end{bmatrix} \right\} \). Is \( S \) linearly dependent?

SOLUTION

Let us transform the following matrix into reduced echelon form

\[
\begin{bmatrix}
1 & 2 & -3 \\
-2 & -3 & 4 \\
5 & 1 & k
\end{bmatrix}
\]

Hence \( S \) is linearly dependent iff \( k = 3 \).
Problems

PROBLEM

Let \( S = \left\{ \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ k \end{bmatrix} \right\} \). Is \( S \) linearly dependent?

SOLUTION

Let us transform the following matrix into reduced echelon form

\[
\begin{bmatrix}
1 & 2 & -3 \\
-2 & -3 & 4 \\
5 & 1 & k
\end{bmatrix}
\sim
\begin{bmatrix}
1 & 2 & -3 \\
0 & 1 & -2 \\
0 & -9 & k + 15
\end{bmatrix}
\]

Hence \( S \) is linearly dependent iff \( k = 3 \).
Problems

PROBLEM

Let \( S = \left\{ \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ k \end{bmatrix} \right\} \). Is \( S \) linearly dependent?

SOLUTION

Let us transform the following matrix into reduced echelon form

\[
\begin{bmatrix}
1 & 2 & -3 \\
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5 & 1 & k
\end{bmatrix} \sim \begin{bmatrix}
1 & 2 & -3 \\
0 & 1 & -2 \\
0 & -9 & k + 15
\end{bmatrix} \sim \begin{bmatrix}
1 & 2 & -3 \\
0 & 1 & -2 \\
0 & 0 & k - 3
\end{bmatrix}
\]

Hence \( S \) is linearly dependent iff \( k = 3 \).
PROBLEM

Let \( S = \left\{ \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ k \end{bmatrix} \right\} \). Is \( S \) linearly dependent?

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Let us transform the following matrix into reduced echelon form

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\begin{bmatrix}
1 & 2 & -3 \\
-2 & -3 & 4 \\
5 & 1 & k
\end{bmatrix} \sim \begin{bmatrix}
1 & 2 & -3 \\
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\end{bmatrix}
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Hence \( S \) is linearly dependant iff \( k = 3 \).
PROBLEM

Let \( S = \left\{ \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ k \end{bmatrix} \right\} \).
PROBLEM

Let \( S = \left\{ \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ k \end{bmatrix} \right\} \). Does \( S \) span \( \mathbb{R}^3 \)?

SOLUTION

Note that the following system has solution for all \( b \) iff \( k \neq 3 \)

\[
\begin{bmatrix}
1 & 2 & -3 & b_1 \\
0 & 1 & -2 & b_2 \\
0 & 0 & k - 3 & b_3
\end{bmatrix}
\]
Problems

PROBLEM

Let \( S = \left\{ \begin{bmatrix} 1 \\ -2 \\ 5 \\ \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \\ \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ k \\ \end{bmatrix} \right\} \). Does \( S \) span \( \mathbb{R}^3 \)?

SOLUTION

Note that the following system has solution for all \( b \) iff \( k \neq 3 \)

\[
\begin{bmatrix}
1 & 2 & -3 & b_1 \\
0 & 1 & -2 & b_2 \\
0 & 0 & k-3 & b_3 \\
\end{bmatrix}
\]

Hence \( S \) spans \( \mathbb{R}^3 \) iff \( k = 3 \).