1. (20 points) Prove the following equalities.
   (a) \(1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}\);
   (b) \(1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2\);
2. (20 points) Prove that for every integers \(a_1, \ldots, a_n\) there are \(k > 0\) and \(\ell \geq 0\) such that \(k + \ell \leq n\) and 
\[
\sum_{i=0}^{\ell} a_{k+i} \text{ is divisible by } n.
\]
3. (10 points) How many 6-digit numbers are there that have the same reminder modulo 2 of all the digits?
4. (20 points) How many pairs of subsets $A, B \subseteq [n]$ are there such that $A \cap B \neq \emptyset$. 