1. (80 points) Check all the correct statements.

- □ There are 6 permutations of \([4]\) with 3 cycles.
- □ The number of different strings you can get by reordering letters in the word aabbc is 30.
- □ The following graph is connected.

\[
\begin{array}{ccc}
A & \rightarrow & B \\
\downarrow & & \downarrow \\
C & \rightarrow & D \\
\end{array}
\]

- □ There are 125 different strings of length 5 over the alphabet with 3 letters.
- □ If you have 26 balls in 5 boxes, then there is a box with at least 6 balls.
- □ There are 6 different surjective functions from \([3]\) to \([3]\).
- □ The following graph has an Euler path.

\[
\begin{array}{ccc}
A & \rightarrow & B \\
\downarrow & & \downarrow \\
C & \rightarrow & D \\
\end{array}
\]

- □ There are 15 variants to put 4 identical balls into 3 different boxes.
2. (10 points) Let $a_n = 2a_{n-1} - a_{n-2}$ for $n \geq 2$, $a_1 = 2$, and $a_0 = 1$. Find a closed formula (no summation signs) for the recurrent sequence $a_n$. 


3. (10 points) Let $i_1, \ldots, i_k \in [n]$ be some different integers. Find a closed formula (no summation signs) for number of permutations $\pi$ such that

$$\pi^{-1}(i_1) \leq \pi^{-1}(i_2) \leq \cdots \leq \pi^{-1}(i_k).$$