1. Elements of $\mathbb{Z}^2$ are colored in black and white, show that there are $x_1, x_2, y_1, y_2 \in \mathbb{Z}$ such that $(x_1, y_1), (x_1, y_2), (x_2, y_1), (x_2, y_2)$ are colored in the same color.
2. Prove the following equality:

\[ \sum_{k=0}^{m} \binom{n+k}{k} = \binom{n + m + 1}{n}. \]
3. Find a closed formula for: \( \sum_{i=0}^{n} i^3 \).
4. Find a recurrent relation for the number of permutations $\pi \in S_n$ such that $\pi^3(x) = x$ for all $x \in [n]$. 