1. Elements of \( \mathbb{Z}^2 \) are colored in black and white, show that there are \( x_1, x_2, y_1, y_2 \in \mathbb{Z} \) such that \( (x_1, y_1), (x_1, y_2), (x_2, y_1), (x_2, y_2) \) are colored in the same color.
2. Prove the following equality:

\[ \sum_{k=0}^{m} \binom{n+k}{k} = \binom{n+m+1}{n}. \]
3. Find a closed formula for: $\sum_{i=0}^{n} i^3$. 
4. Find a recurent relation for the number of permutations $\pi \in S_n$ such that $\pi^3(x) = x$ for all $x \in [n]$. 