1. (a) Find a closed formula (no summation signs) for the expression $S(n, n - 1)$.

(b) Find a closed formula (no summation signs) for the expression $S(n, 3)$. 

2. Find a closed formula (no summation signs) for the expression \( \sum_{k=0}^{n} (-1)^k \binom{2n}{2k} \).
3. We colored all points of $\mathbb{R}^2$ with integer coordinates by one of six colors. Prove that there is a rectangle whose vertices are monochromatic. Can we make the statement stronger by limiting the size of the purported monochromatic rectangle?
4. Prove the following inequality for all integers $n$ and real $x \geq -1$, $(1 + x)^n \geq 1 + nx$. 