1. We call a partition \( \{P_1, \ldots, P_k\} \) of \( [n] \) nice iff \((j + 1) \notin P_i \) for every \( i \in [k] \) and \( j \in P_i \).
Prove that number of nice partitions is equal to \( B(n - 1) \).

Solution: Define \( S_n = \) the set of all partitions of \( [n] \) and \( M_n = \) the set of all nice partitions of \( [n] \).
We are going to construct a bijection
\[
f : S_{n-1} \rightarrow M_n.
\]
Notice that we can obtain every nice partition of \( [n] \) by:

1. First adding a singleton block \( \{n\} \) to any partition of \( [n] \).
2. Then, in each block of that partition of \( [n - 1] \), we locate consecutive integers \( i, i + 1, \ldots, i + j \)
   and if \( j \) is odd place every other integer, i.e \( i, i + 2, i + 4, \ldots, i + j - 1 \), into the block with \( n \),
   if \( j \) is even we put \( i + 1, \ldots, i + j - 1 \) into the block with \( n \). We do this for each consecutive
   sequence of each block of each partition of \( [n - 1] \) to obtain all possible nice partitions of \( [n] \).

Note that the resulting partition is nice, since \( i + j \leq n - 1 \), hence \( i + j - 1 < n - 1 \).
For each nice partition of \( [n] \) obtained this way we can have an inverse transformation \( f^{-1} \) that
takes every nice partition of \( [n] \) and gives the corresponding original partition of \( [n - 1] \) by:

1. Taking every element except \( n \) in the block that contains \( n \) and placing each element \( i \), in
   order, into a block that contains \( i - 1 \).
2. We then remove the block of \( n \) from our partition resulting in a partition of \( [n - 1] \).

It can be seen that applying \( f^{-1} \) to every partition of \( M_n \) gives us every partition of \( S_n \). Thus \( f \)
forms a bijection between \( M_n \) and \( S_{n-1} \).
2. How many different 6-digit numbers have sum of their digits at most 47?

**Solution:** Let $F$ denote the number of 6-digit numbers whose sum is less than 48. Note that number $G$ of 6-digit numbers with sum of digits at least 48 is equal to $9 \cdot 10^5 - F$.

Let us find $G$. Note that the maximal sum of digits of a 6-digit number is $6 \cdot 9 = 54$. Hence, in order to transform $999999$ into another number with sum of digits at least 48 we need to substract $54 - 48 = 6$ from the digits of $999999$; i.e. we need to put at most $54 - 48 = 6$ balls into 6 boxes (each digit corresponds to a box).

Hence, $G = \sum_{i=0}^{6} \binom{6+i-1}{i} = \sum_{i=0}^{6} \binom{5+i}{5} = \binom{12}{6} = 924$. Thus $F = 9 \cdot 10^5 - 924 = 899076.$
3. How many ways to put \( n \) indistinguishable balls into \( k \) different boxes if we have to put at least \( a_i \) balls into the box with number \( i \).

**Solution:** Let \( i \in [k] \) and \( a_i \) denote the minimum number of balls the \( i \)th box contains for all \( a_1, \ldots, a_k \). So we can place \( a_i \) balls into the \( i \)th box for all \( i \). Let \( j = a_1 + a_2 + \cdots + a_k \) then we have \( n - j \) balls left to place into \( k \) boxes. This is then a weak composition problem, thus the solution is \( \binom{n-j+k-1}{k-1} \).