1. What is the minimal number $k$ such that for every subset $S$ of $\{1, \ldots, 20\}$, if $|S| = k$, then there are $x, y \in S$ such that $x + y$ is divisible by 21.

**Solution:** Let us prove that the answer is 11. If $k < 11$, then $S = \{1, \ldots, k\}$ does not contain $x$ and $y$ such that $x + y$ is divisible by 21 (since the maximum of $x + y$ is $2k - 1 \leq 20 - 1 = 19$).

Let us prove that if $k \geq 11$ then there are such $x$ and $y$. Note that there are 10 pairs $1 + 20, \ldots, 10 + 11$ such that their sum is equal to 21. Hence, by the pigeonhole principle, there are two elements of $S$ such that they belong to the same pair; i.e. their sum is equal to 21.