1. (10 points) Find a closed formula (no summation signs) for the expression $S(n, 4)$.

**Solution:** The solution of this problem is similar to the solution of the problem for $S(n, 3)$.

Let us count number of functions $f$ such that $f : [n] \to 4$ is not a surjection. If $f$ is a surjection, then range of $f$ consists of at most 3 elements. There are $\binom{4}{3}$ variants to choose a set $S$ of these elements there are and $3^n$ different functions with codomain $S$. Hence, the first approximation of the answer is $\binom{4}{3}3^n$.

However, we count twice all the functions with range consisting of two elements and there are $\binom{4}{2}2^n$ functions like this. Hence, the second approximation is $\binom{4}{3}3^n - \binom{4}{2}2^n$.

Finally, we subtracted twice functions with range consisting of one element, and there are 4 functions like this. As a result, there are $\binom{4}{3}3^n - \binom{4}{2}2^n + 4$ nonsurjective functions $f : [n] \to 4$ and the final answer is

$$4^n - \binom{4}{3}3^n + \binom{4}{2}2^n - 4 \over 4!.$$