

# MATH 103A – Midterm 1 Study Guide

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First, let me warn you that this is by no means a complete list of problems, or topics. Just highlights. The first thing you should do when preparing for the exam is to go through your notes, the relevant chapters of the book and the homework problems. As well as the suggested extra problems for each chapter. If you have trouble with some of the topics encountered so far, take the book and solve more problems related to that topic until you *really* understand how and why things work.

## Topics

modular arithmetic, prime numbers, injectivity, surjectivity, bijectivity  
groups (definition, examples, etc...); abelian groups; subgroups; centralizer of an element or set; center of a group; cyclic groups and subgroups; permutation groups  
order of a group, subgroup, element

## Concept review

1. How do you add and multiply  $(\text{mod } n)$ ?
2. How do you decompose a number into primes?
3. What is the gcd of two integers? How do you find it (algorithm and prime decomposition)?
4. What is the lcm of two integers? How do you find it?
5. GCD can be written as a linear combination of the two integers.
6. What is a group?
7. Examples of groups: review all the examples discussed in class. You should become really well acquainted with  $Z_n, U(n), D_n, S_n$ , and the matrix groups  $GL(2, \mathbb{R}), SL(2, \mathbb{Z}), GL(2, Z_n), SL(2, Z_n)$ .
8. What is an abelian group?
9. What is a subgroup?
10. Subgroup tests (Theorems 3.1, 3.2)
11. What is the subgroup generated by an element?
12. What is a cyclic group? A cyclic subgroup?
13. What is the order of an element?
14. Theorem 4.1 and its corollaries.
15. Theorem 4.2 and its corollaries.
16. Theorem 4.3 and its corollary.

17. Theorem 4.4.
18. What is the center of a group? How do you compute it?
19. What is the centralizer of an element in a group? How do you compute it? Take your favorite non-abelian group and compute the order of at least two elements.
20. What is  $Z_n$ ? With what operation does it become a group? How many elements does it have? What is its center?
21. What is  $U(n)$ ? With what operation does it become a group? How many elements does it have? What is its center?
22. What is the Euler  $\varphi$  function? How do you compute it? (E.g.  $\varphi(960) = ?$ )
23. What is  $D_n$ ? How many elements does it have? What is its center?
24. What is  $S_n$ ? How many elements does it have?
25. What is a cycle? What is its order?
26. How do you decompose a permutation as a product of disjoint cycles?
27. Theorems 5.2 and 5.3.
28. What is the order of a product of 17 disjoint cycles?