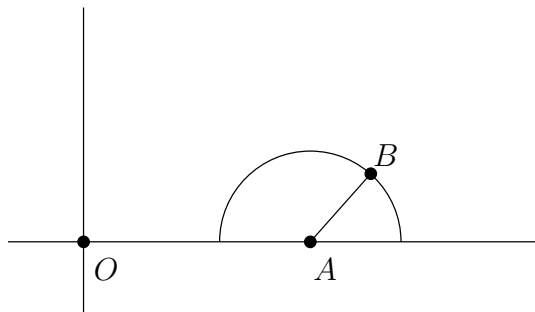# MATH 20C – MIDTERM 1
SOLUTIONS TO PRACTICE PROBLEMS, PART 2

ALINA BUCUR

Problem 12:

(a) $\overrightarrow{AB} = \langle \cos t, \sin t \rangle$ and $\overrightarrow{OA} = \langle 10t, 0 \rangle$, so $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \langle 10t + \cos t, \sin t \rangle$.

The rear bumper is reached at time $t = \pi$ and the position of B is $(10\pi - 1, 0)$.

(b) $\vec{v}(t) = \langle 10 - \sin t, \cos t \rangle$, so

$$|\vec{v}|^2 = (10 - \sin t)^2 + \cos^2 t = 100 - 20 \sin t + \sin^2 t + \cos^2 t = 101 - 20 \sin t.$$

The speed is then given by $|\vec{v}| = \sqrt{101 - 20 \sin t}$.

The speed is smallest when $\sin t$ is largest i.e. $\sin t = 1$. It occurs when $t = \pi/2$. At this time, the position of the bug is $(5\pi, 1)$.

The speed is largest when $\sin t$ is smallest; that happens at the times $t = 0$ or π for which the position is then $(0, 0)$ and $(10\pi - 1, 0)$.