MATH 20C Lecture 26 - Monday, March 10, 2014

More triple integrals. Discussed problems 32, 34, 35 from the study guide.

MATH 20C Lecture 27 - Wednesday, March 12, 2014

Review - part I

Topics:

- vectors: quantities that have length and direction
- operation with vectors (addition, subtraction, multiplication by scalars, dot product, cross product);
- determinants, area, volume;
- equations of lines and planes in 3-space (normal vectors)
- decomposing a vector into a components along specified direction (projection, $\vec{v}_{\parallel}$ and $\vec{v}_{\perp}$)
- parametric equations
- calculus with vectors: limits, differentiation (product rules, chain rule), integration;
- velocity and acceleration vectors; speed and arc length
- tangent line to a trajectory
- Functions of several variables, contour plots.
- Partial derivatives, gradient; approximation formulas, tangent planes, directional derivatives.
- higher order partial derivatives
- chain rule, change of variables, implicit differentiation

MATH 20C Lecture 28 - Friday, March 14, 2014

Review - part II

- Min/max problems: critical points, second derivative test, checking boundary. (least squares won't be on the exam)
- Min/max for non-independent variables: Lagrange multipliers. *Second derivative test does NOT work! Plug in values or use geometry.*
- Double integrals: drawing picture of region, taking slices to set up the iterated integral

- Same in polar coordinates (recall that $dA = dx\,dy = r\,dr\,d\theta$).

  Example: Write in polar coordinates $\iint_R f(x,y)dA$ for R the disk of radius 1 centered at $(1,0)$. (Picture drawn)

  Since we are at the right of the y-axis, get $-\pi/2 \le \theta \le \pi/2$. For each, theta can use geometry to see $0 \le r \le 2\cos\theta$. Or can deduce the same with algebra, as follows. The equation of the circle is $(x-1)^2 + y^2 = 1$. Substitute polar coordinates ($x = r\cos\theta, y = r\sin\theta$) and get $r(r - 2\cos\theta) = 0$. Hence $r = 0$ and $r = 2\cos\theta$ are the two endpoints of the ray at θ. Get

  $$\int_{-\pi/2}^{\pi/2} \int_0^{2\cos\theta} f(r,\theta) r\,dr\,d\theta.$$

- Triple integrals in rectangular and cylindrical coordinates ($dV = dA\,dz$): setup, pictures.

- Triple integrals in spherical coordinates ($dV = \rho^2 \sin\phi d\rho d\phi d\theta$): setup, pictures.

- For evaluation, need to know: usual basic integrals (e.g. $\int \frac{dx}{x}$); integration by substitution (e.g. $\displaystyle\int_0^1 \frac{2tdt}{\sqrt{1+t^2}} = \int_1^2 \frac{du}{\sqrt{u}}$ by setting $u = 1 + t^2$), integration by parts. DO NOT need to know: complicated trigonometric integrals (e.g. $\int \cos^4\theta d\theta$).

- Applications: area, volume, mass, average and weighted average of a function, center of mass.

We have not discussed all the topics in class due to lack of time. They are all equally important.

This is it. Good luck on the exam!