HOMEWORK 5

DUE 11 FEBRUARY 2015

SHOW ALL YOUR WORK.

1. Find all positive integer solutions to $x^2 + 12 = y^4$.

2. Find all positive integer solutions to $x^3 + y^3 = 20$.

3. Show that a real number A is rational if and only if its continued fraction expansion is finite.

4. Find the value of each of the following periodic continued fractions. Express your answer in the form

$$\frac{a + b\sqrt{d}}{c},$$

where a, b, c, d are integers.

(a) $[\overline{1}] = [1, 1, 1, \ldots]$

(b) $[\overline{1, 2, 3}] = [1, 2, 3, 1, 2, 3, 1, 2, 3, \ldots]$

(c) $[1, \overline{2, 3}] = [1, 2, 3, 2, 3, 2, 3, \ldots]$

5. For each of the following numbers find their (periodic) continued fraction. What's the period in each case?

(a) $\dfrac{16 - \sqrt{3}}{11}$

(b) $\dfrac{1 + 2\sqrt{5}}{3}$

(c) $\dfrac{1+\sqrt{5}}{2}$.