HOMEWORK 1

DUE 13 APRIL 2015

SHOW ALL YOUR WORK.

1. Show that improper equivalence is not an equivalence relation.

2. Show that equivalent forms represent the same numbers. Show that the same holds for proper representations. Here you need to show that, given $m \in \mathbb{Z}$, the equation $f(x, y) = m$ has the same number of solutions in integers for all the forms in the same equivalence class. Similarly for proper representations.

3. Show that any form equivalent to a primitive forms is itself primitive. *Hint: Use the previous exercise.*

4. Suppose $f(x, y)$ is a bqf with discriminant D and $g(x, y)$ is a bqf with discriminant D'. Suppose further that

$$f(x, y) = g(\alpha x + \beta y, \gamma x + \delta y) \text{ for some } \alpha, \beta, \gamma, \delta \in \mathbb{Z}.$$

Show that

$$D = (\alpha\delta - \beta\gamma)^2 D'.$$

5. Prove that if two bqf's are equivalent, then they have the same discriminant. *Hint: Use the previous exercise.*

6. Show that the following bqfs are properly equivalent:

(a) $2x^2 + 2xy + 3y^2$ and $2x^2 - 2xy + 3y^2$;

(b) $x^2 + 4y^2$ and $x^2 + 2xy + 5y^2$.

7. Show that the following bqfs are not properly equivalent. Are they equivalent?

(a) $x^2 + 5y^2$ and $2x^2 + 2xy + 3y^2$;

(b) $x^2 + 4y^2$ and $x^2 + 2xy + 3y^2$.

8. Prove that if $f(x, y) = ax^2 + bxy + cy^2$ is a primitive, positive definite bqf with $|b| \leq a \leq c$, then

$$f(x, y) \geq (a - |b| + c) \min x^2, y^2.$$

Did you use the fact that $|b| \leq a \leq c$ in the proof? Was it essential? If so, find a counterexample when the condition is removed.