# HOMEWORK 2

DUE 20 APRIL 2015

**SHOW ALL YOUR WORK.**

**1.** Find all the reduced forms of discriminant $D$ and compute the class number $h(D)$ for each of the following discriminants.
(a) $D = -20$
(b) $D = -56$
(c) $D = -28$
(d) $D = -15$

**2.** Let $p$ be a prime number which is represented by forms $f(x, y)$ and $g(x, y)$ of discriminant $D$.
(a) Show that $f(x, y)$ and $g(x, y)$ are equivalent.
*Hint: use Lemma 3.15 and examine the middle coefficient modulo p.*

(b) If $f(x, y) = x^2 + ny^2$ and $g(x, y)$ is reduced, show that $f(x, y) = g(x, y)$.

**3.** Consider the binary quadratic form $f(x, y) = ax^2 + bxy + cy^2$ and assume that it is primitive.

(a) Given a prime $p$, prove that at least one of $f(1, 0), f(0, 1)$ and $f(1, 1)$ is relatively prime to $p$.

(b) Given an integer $M$, show that $f(x, y)$ properly represents numbers relatively prime to $M$.
*Hint: use (a) and the Chinese Remainder Theorem.*

**4.** Prove that if $p \neq 2, 7$ is a prime number, then

$$p = x^2 + 14y^2 \text{ or } 2x^2 + 7y^2 \iff p \equiv 1, 9, 15, 23, 25, 39 \pmod{56}$$

and

$$p = 3x^2 \pm 2xy + 5y^2 \iff p = 3, 5, 13, 19, 27, 45 \pmod{56}.$$

**5.** Prove that $p = x^2 + 10y^2 \iff p \equiv 1, 9, 11, 19 \pmod{40}$.

**6.** Prove that $p = x^2 + 21y^2 \iff p \equiv 1, 25, 37 \pmod{84}$.

**7.** Work out the genus theory of Theorem 3.30 for the following discriminants.
(a) $D = -15$

(b) $D = -24$