

HOMEWORK 4

DUE 11 MAY 2015

SHOW ALL YOUR WORK.

1. Show that

$$\sum_{p \leq x} \frac{1}{p} = \log \log x + O(1).$$

2. Prove by partial summation that

$$\psi(x) \sim x \iff \pi(x) \sim \frac{x}{\log x}.$$

Hint: prove first that

$$\theta(x) \sim x \iff \pi(x) \sim \frac{x}{\log x}.$$

3. Using equation (6.2) from the notes, show that

$$\theta(x) - \theta\left(\frac{x}{2}\right) \geq \frac{x}{3} \log 2 + O(\sqrt{x} \log^2 x).$$

4. Using the previous exercise, prove that there exists a constant $A > 0$ such that

$$\theta(x) > Ax$$

for large enough x .5. **(Extra credit)** Show that there exist positive constants A, B such that $B/A \leq 2$ and $Ax < \theta(x) < Bx$ for x large enough.