MATH 20C **Lecture C**

Final Exam **December 8, 2010**

Name: ______________________________

PID: ______________________________

TA: ______________________________

Section #: __________ Section time: ______________

There are 14 pages and 11 questions, for a total of 200 points.

No notes, no calculators, no books.

Please turn off all electronic devices.
Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. Unless otherwise stated, show all your work for full credit. To maximize credit, cross out incorrect work.

Good luck! ☺

Question:	1	2	3	4	5	6	7	8	9	10	11	Total
Points:	20	35	20	10	25	10	10	15	15	15	25	200
Score:												

1. For each of (a)–(d) below: If the statement is true, write TRUE. If the statement is false, write FALSE. (**Please do NOT use the abbreviations T and F.**) No explanations are required in this problem.

   (a) (5 points) $\vec{A} \cdot (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{B}) \times \vec{C}$ for all vectors $\vec{A}, \vec{B}, \vec{C}$.

   (b) (5 points) If $f(x, y)$ is a differentiable function and $\nabla f(P)$ is perpendicular to the unit vector $\hat{\mathbf{u}}$, then $\hat{\mathbf{u}}$ is parallel to the tangent plane at P to the graph of f.

   (c) (5 points) If $f(x, y)$ is continuous, then

$$\int_{\pi/2}^{\pi/2} \int_0^1 f(r\cos\theta, r\sin\theta) r\, dr\, d\theta = \int_{-1}^{1} \int_0^{\sqrt{1-y^2}} f(x, y)\, dx\, dy.$$

   (d) (5 points) The volume of the unit sphere is given by

$$4 \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} dz\, dy\, dx.$$

2. The top extremity of a ladder of length L rests against a vertical wall, while the bottom is being pulled away.

   (a) (15 points) Find parametric equations for the midpoint P of the ladder, using as parameter the angle θ between the ladder and the horizontal ground.

   (b) (5 points) Compute the velocity vector of P.

(c) (5 points) Compute the acceleration vector of P.

(d) (10 points) Compute the length of the trajectory described by P between $\theta = \pi/4$ to $\theta = 0$.

3. Let $P = (1, 0, 1), Q = (0, 3, 1)$ and $R = (0, 1, 4)$.
   (a) (7 points) Find the area of the triangle in space PQR.
   (b) (5 points) Find the plane through P, Q and R expressed in the form $ax+by+cz = d$.
   (c) (8 points) Is the line through $(1, 2, 3)$ and $(2, 2, 0)$ parallel to this plane? Explain why or why not.

4. (10 points) Find the Lagrange multiplier equations for the point of the surface

$$\sin(x^2 - yz^2) + xyz = 300$$

at which $f(x, y, z) = e^{xyz}$ is largest. DO NOT SOLVE.

5. (25 points) Let $f(x, y) = 10x + 4y - 3$. Find the global min and global max in the triangle with vertices $P = (1, 1), Q = (1, 2), R = (2, 1)$.

6. (10 points) Let $w = f(u, v)$, where $u = xy^2$ and $v = (x^2 + 3)/y$. Using the chain rule, express $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$ in terms of x, y, f_u and f_v.

7. Suppose that the function $T = x^2 + 2xy + 2y^2$ gives the temperature at each point of the plane, and that one starts at the point $(2, 1)$.

   (a) (5 points) In which direction should one go to obtain the most rapid increase in T? Express your answer as a unit vector.

   (b) (5 points) Approximately how far in that direction should one go to get an increase of 0.1 in T?

8. (a) (5 points) Sketch the region R in the xy-plane corresponding to the iterated integral

$$\int_0^1 \int_{x^2}^{\sqrt{x}} y^2 \, dy \, dx.$$

(b) (10 points) Set up the integral from part (a) in polar coordinates. Give the integrand and bounds, but DO NOT EVALUATE.

9. Set up an iterated integral giving the volume of the solid enclosed by the planes $z = 0$ and $z = x + y + 3$ and the cylinders $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$

   (a) (5 points) in rectangular coordinates

   (b) (10 points) in cylindrical coordinates

10. (a) (5 points) Sketch the region R in the xy-plane corresponding to the iterated integral

$$\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\, dx\, dy.$$

(b) (10 points) Interchange the order of integration and compute the integral from part (a).

11. (25 points) Let R be the region in the first octant $x, y, z \geq 0$ bounded by $z = 1 - y^2$ and $y = 1 - x$. Find the mass of the solid in the shape of R with mass-density $\rho(x, y, z) = z$.

This is the end. Enjoy your winter break!

Continuation of work on problem number ________
(Detach and recycle this page if it is not part of your solutions.)