MATH 20C – MIDTERM 1
ANSWERS TO PRACTICE PROBLEMS

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This are just answers, not completely written out solutions!

Problem 2: (a) $\frac{\sqrt{6}}{2}$

(b) $x + y + 2z = 3$.

(c) $(0, 1, 1)$.

Problem 3: (a) $\overrightarrow{QP} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}}, \quad \overrightarrow{QR} = -2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

(b) $\frac{4}{\sqrt{65}}$

(c) $\frac{9}{\sqrt{130}}$

Problem 4: (a) $7/2$

(b) $6x + 3y + 2z = 11$

(c) Line has direction $\langle 1, 0, -3\rangle$ and $\vec{N} \cdot \langle 1, 0, -3\rangle = 0$. Therefore line $\perp \vec{N}$ and so it is parallel to the plane.

Problem 5: $\vec{L}(u) = \langle -1, 1, 1 + u\rangle$

Problem 6: $\pi/4$

Problem 7: $\vec{v} = \langle -3\sin t, 3\cos t, 1\rangle, \quad \vec{a} = \langle -3\cos t, -3\sin t, 0\rangle, \quad \text{speed} = \sqrt{10}$

Problem 8: (a) $\vec{N} = \langle 4, -3, -2\rangle$

(b) $\vec{N} \cdot \vec{r}(t) = 6$.

(c)

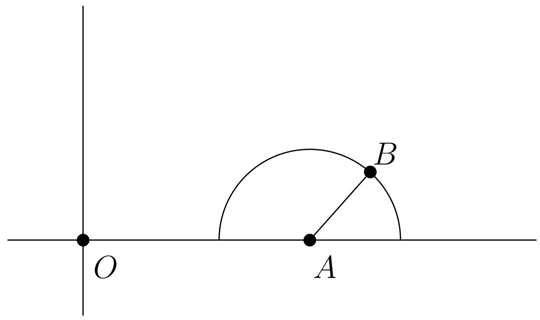
$$\frac{d}{dt}\left(\vec{N} \cdot \vec{r}(t)\right) = 0 \;=: \frac{d\vec{N}}{dt} \cdot \vec{r}(t) + \vec{N} \cdot \frac{d\vec{r}}{dt} = 0 \;=: \vec{N} \cdot \frac{d\vec{r}}{dt} = 0 \;=: \vec{N} \cdot \vec{v}(t) = 0 \;=: \vec{N} \perp \vec{v}(t).$$

Problem 9: $\vec{r}(t) = \langle t, \cos t, t\sin t\rangle$

Problem 10: $\vec{L}(u) = \langle 0, 3, 2u \rangle, \quad \vec{a}(0) = \langle 2, -3, 0 \rangle$

Problem 11: $t^3 + 2t$

Problem 12:



(a) $\overrightarrow{AB} = \langle \cos t, \sin t \rangle$ and $\overrightarrow{OA} = \langle 10t, 0 \rangle$, so $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \langle 10t + \cos t, \sin t \rangle$.

The rear bumper is reached at time $t = \pi$ and the position of B is $(10\pi - 1, 0)$.

(b) $\vec{v}(t) = \langle 10 - \sin t, \cos t \rangle$, so

$$|\vec{v}|^2 = (10 - \sin t)^2 + \cos^2 t = 100 - 20 \sin t + \sin^2 t + \cos^2 t = 101 - 20 \sin t.$$

The speed is then given by $|\vec{v}| = \sqrt{101 - 20 \sin t}$.
The speed is smallest when $\sin t$ is largest i.e. $\sin t = 1$. It occurs when $t = \pi/2$. At this time, the position of the bug is $(5\pi, 1)$.
The speed is largest when $\sin t$ is smallest; that happens at the times $t = 0$ or π for which the position is then $(0, 0)$ and $(10\pi - 1, 0)$.