MATH 20C
Exam 1

Lecture C
October 18, 2010

Name: SOLUTIONS (yellow version)

PID: ______________________

TA: ______________________

Section #: ________ Section time: ______________

There are 6 pages and 4 questions, for a total of 60 points.

No notes, no calculators, no books.

Please turn off all electronic devices.
Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. Unless otherwise stated, show all your work for full credit. To maximize credit, cross out incorrect work.

Good luck! ☺

Question:	1	2	3	4	Total
Points:	20	20	10	10	60
Score:					

MATH 20C, Lecture C Name: ______________________

Exam 1, 10/18/2010 Section #: Section time:

1. Consider the points in space $P = (1, 2, -1), Q = (2, 1, 1), R = (1, -1, 2)$.

   (a) (3 points) Compute the vector $\overrightarrow{PQ}$.

   $\overrightarrow{PQ} = \langle 1, -1, 2 \rangle$

   (b) (5 points) Find a normal vector to the plane through P, Q and R.

   $\overrightarrow{PR} = \langle 0, -3, 3 \rangle \qquad \overrightarrow{QR} = \langle -1, -2, 1 \rangle$

   $$\text{normal vector} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 0 & -3 & 3 \end{vmatrix} = \begin{vmatrix} -1 & 2 \\ -3 & 3 \end{vmatrix} \hat{i} - \begin{vmatrix} 1 & 2 \\ 0 & 3 \end{vmatrix} \hat{j} + \begin{vmatrix} 1 & -1 \\ 0 & -3 \end{vmatrix} \hat{k}$$

   $$= 3\hat{i} - 3\hat{j} - 3\hat{k} = \langle 3, -3, -3 \rangle$$

   (c) (6 points) Compute the area of the triangle in space PQR.

   $$\text{Area } \Delta = \frac{1}{2} |\overrightarrow{PQ} \times \overrightarrow{PR}| = \frac{1}{2}\sqrt{9+9+9} = \frac{\sqrt{27}}{2} = \frac{3\sqrt{3}}{2}$$

   (d) (6 points) Find the cosine of the angle at P of the triangle PQR.

   $$\cos\theta = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{|\overrightarrow{PQ}|\,|\overrightarrow{PR}|} = \frac{3+6}{\sqrt{6}\,\sqrt{18}} = \frac{\sqrt{3}}{2}$$

2. Consider a moving point whose trajectory is given by $\vec{r}(t) = \langle \cos 3t, \sin 3t, 4t \rangle$.

(a) (3 points) Find the velocity vector $\vec{v}(t)$.

$\vec{v}(t) = \vec{r}\,'(t) = \langle -3\sin 3t, 3\cos 3t, 4 \rangle$

(b) (7 points) Write down a parametric equation for the tangent line $\vec{L}(\theta)$ at the point $P = (1, 0, 0)$.

$0 = 4t \Rightarrow t = 0$

$\vec{v}(0) = \langle 0, 3, 4 \rangle$

$\vec{L}(\theta) = \langle 1, 0, 0 \rangle + \theta \langle 0, 3, 4 \rangle$

$= \langle 1, 3\theta, 4\theta \rangle$

(c) (2 points) What is the speed of the moving point at P?

$\|\vec{v}(0)\| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

(d) (3 points) Find the acceleration $\vec{a}(t)$.

$$\vec{a}(t) = \langle -9\cos 3t, -9\sin 3t, 0 \rangle$$

(e) (5 points) Find the length travelled by the moving point along the trajectory between $t_1 = 0$ and $t_2 = \pi$.

$$S = \int_0^{\pi} \|\vec{v}(t)\|\, dt = \int_0^{\pi} \sqrt{9\sin^2 3t + 9\cos^2 3t + 16}\, dt$$

$$= \int_0^{\pi} \sqrt{9+16}\, dt = \int_0^{\pi} 5\, dt$$

$$= 5t \Big|_0^{\pi} = 5\pi$$

3. Consider the plane $x - y + 2z = -2$.

(a) (3 points) Find a normal vector to this plane.

$n = \langle 1, -1, 2 \rangle$

(b) (7 points) Find the intersection of this plane with the line parallel to the vector $\langle 1, -1, 1 \rangle$ and passing through the point $P = (1, 1, 1)$.

Eq. of line

$r(t) = \langle 1,1,1 \rangle + t \langle 1,-1,1 \rangle = \langle 1+t, 1-t, 1+t \rangle$

$(1+t) - (1-t) + 2(1+t) = -2$

$2 + 4t = -2$

$t = -1$

Point of intersection

$r(-1) = \langle 0, 2, 0 \rangle$

4. (10 points) Find a parametric equation for the curve $\vec{r}(t)$ given by the intersection of

$$x + 2y^2 = 5 \text{ and } x - 2y^2 + z + 2 = 0.$$

Sol I $y = t \Rightarrow x = 5 - 2t^2$

$z = -2 - x + 2y^2 = 4t^2 - 7$

$\vec{r}(t) = \langle 5 - 2t^2, t, 4t^2 - 7 \rangle$

Sol II $x = t \Rightarrow y^2 = \frac{5-t}{2} \Rightarrow y = \pm\sqrt{\frac{5-t}{2}}$

$z = -2 - x + 2y^2 = 3 - 2t$

$\vec{r}_1(t) = \langle t, \sqrt{\frac{5-t}{2}}, 3-2t \rangle$ AND $\vec{r}_2(t) = \langle t, -\sqrt{\frac{5-t}{2}}, 3-2t \rangle$

(need 2 functions)

Sol III $z = t \Rightarrow x - 2y^2 = -2 - t$

$$\begin{array}{r} x + 2y^2 = 5 \\ \hline 2x = 3 - t \end{array} \Rightarrow x = \frac{3-t}{2}$$

$y^2 = \frac{1}{2}(5 - x) = \frac{1}{2} \cdot \frac{t+7}{2} \Rightarrow y = \pm \frac{\sqrt{t+7}}{2}$

$\vec{r}_1(t) = \langle \frac{3-t}{2}, \frac{\sqrt{t+7}}{2}, t \rangle$ AND $\vec{r}_2(t) = \langle \frac{3-t}{2}, -\frac{\sqrt{t+7}}{2}, t \rangle$

(need 2 functions)