MATH 20C Lecture 1 - Thursday, October 2, 2014

Handout: syllabus

Vectors

A vector (notation: $\vec{A}$) has a direction, and a length ($|\vec{A}|$). It is represented by a directed line segment, or arrow. In a coordinate system it's expressed by components. For instance, in space, $\vec{A} = \langle a_1, a_2, a_3\rangle = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$. (Recall in space x-axis points to the lower-left, y-axis to the right, z-axis up.) It tells me in which direction and how far to move.

Scalar multiplication

Just scale the vector (keep the direction, change the length).

Formula for length

Drew the vector $\langle 1, 2, 3\rangle$ and asked for its length. Most students got the right answer ($\sqrt{14}$).

Explained how why $|\vec{A}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$ by reducing to the Pythagorean theorem in the plane. Namely, we first drew a picture showing $\vec{A}$ and its projection on the xy-plane, then derived $\vec{A}$ from the length of the projection and the Pythagorean theorem (applied twice).

Vector addition

Drew a picture of the parallelogram with sides $\vec{A}$ and $\vec{B}$ and showed how the diagonals are $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$. Addition works componentwise:

$$\vec{A}+\vec{B} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3\rangle$$

Multiplication by scalars

Same direction or opposite, just length changes. Drew pictures of $\vec{A}, 2\vec{A}, \frac{1}{2}\vec{A}, -\vec{A}$.

$$10\vec{A} = \langle 10a_1, 10a_2, 10a_3\rangle$$

So, for subtraction

$$\vec{A}-\vec{B} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3\rangle$$

And indeed $\vec{A} = \langle a_1, a_2, a_3\rangle = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$ in our earlier example.