

MATH 20C Lecture 16 - Tuesday, December 9, 2014

Review topics

- vectors: quantities that have length and direction
- operation with vectors (addition, subtraction, multiplication by scalars, dot product, cross product);
- determinants, area, volume;
- equations of lines and planes in 3-space (normal vectors)
- decomposing a vector into a components along specified direction (projection, \vec{v}_{\parallel} and \vec{v}_{\perp})
- parametric equations
- calculus with vectors: limits, differentiation (product rules, chain rule), integration;
- velocity and acceleration vectors; speed and arc length
- tangent line to a trajectory
- Functions of several variables, contour plots.
- Partial derivatives, gradient; approximation formulas, tangent planes, directional derivatives.
- higher order partial derivatives
- chain rule, change of variables, implicit differentiation
- recover function from gradient
- Min/max problems: critical points, second derivative test, checking boundary. (least squares won't be on the exam)
- Min/max for non-independent variables: Lagrange multipliers. *Second derivative test does NOT work! Plug in values or use geometry.*

MATH 20C Lecture 17 - Thursday, December 11, 2014

Review (continued)

- Double integrals: drawing picture of region, taking slices to set up the iterated integral
- Same in polar coordinates (recall that $dA = dx dy = r dr d\theta$).

Example: Write in polar coordinates $\iint_R f(x, y) dA$ for R the disk of radius 1 centered at $(1, 0)$. (Picture drawn)

Since we are at the right of the y -axis, get $-\pi/2 \leq \theta \leq \pi/2$. For each, theta can use geometry to see $0 \leq r \leq 2 \cos \theta$. Or can deduce the same with algebra, as follows. The equation of

the circle is $(x - 1)^2 + y^2 = 1$. Substitute polar coordinates ($x = r \cos \theta, y = r \sin \theta$) and get $r(r - 2 \cos \theta) = 0$. Hence $r = 0$ and $r = 2 \cos \theta$ are the two endpoints of the ray at θ . Get

$$\int_{-\pi/2}^{\pi/2} \int_0^{2 \cos \theta} f(r, \theta) r \, dr \, d\theta.$$

- Triple integrals in rectangular and cylindrical coordinates ($dV = dA \, dz$): setup, pictures.
- For evaluation, need to know: usual basic integrals (e.g. $\int \frac{dx}{x}$); integration by substitution (e.g. $\int_0^1 \frac{2t \, dt}{\sqrt{1+t^2}} = \int_1^2 \frac{du}{\sqrt{u}}$ by setting $u = 1 + t^2$), integration by parts. DO NOT need to know: complicated trigonometric integrals (e.g. $\int \cos^4 \theta \, d\theta$).
- Applications: area, volume, mass, average and weighted average of a function, center of mass.

Next we discussed problems 30, 34, 35, 38, 39, 41 from the study guide.

This is it. Good luck on the exam!