

Mathematics 20D
Second Midterm, November 24, 2012

Maximum score on this midterm is 50 points.

Write your solutions in the space provided. In case need more space than provided use the back-page.

Important: Show your reasoning! Answers without explanations will receive no credit, even if they are correct!

Full name: _____

Section: _____

Student ID: _____

Signature: _____

Problem 1. _____

Problem 2. _____

Problem 3. _____

Total: _____

Problem 1. (15 points) Solve the following differential equation:

$$y'' + 4y' + 4y = t^{-2}e^{-2t}, \quad y(1) = e^{-2}, y'(1) = -3e^{-2}, \quad t > 0.$$

Problem 2. (17 points) Consider the following system:

$$\mathbf{x}' = \begin{pmatrix} \alpha & 2 \\ -5 & -1 \end{pmatrix} \mathbf{x}$$

where α is a real parameter.

i) For which value of α , does the origin become a center? (trajectories do not move away or towards the origin).

ii) For the value of α found in part i), find the general solution of the system. Sketch the phase portrait: emphasize the shape of trajectories and their direction (by drawing an arrow on the trajectory).

iii) What is the full range of α for which the origin is asymptotically stable (that means all the trajectories converge to the origin as $t \rightarrow \infty$).

Problem 3. (18 points) i) Find the general solution for the following system of equations

$$\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} \mathbf{x}.$$

ii) Compute e^{At} for

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}.$$

iii) Solve the equation in part i) for initial data

$$\mathbf{x}(0) = \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

Extra Page.