MATH 20D
Exam 1 v2

Lecture A
April 21, 2015

Name: SOLUTIONS

PID: ____________________

TA: ____________________

Section #: ________ Section time: ____________

Seat: ____________________

There are 6 pages and 4 questions, for a total of 100 points.

No calculators, no electronic devices, no books, no notes, except for the one 8.5in×11in sheet of notes.

Please turn off all electronic devices.

Answer the questions in the spaces provided on the question sheets. Unless otherwise stated, show all your work for full credit. To maximize credit, cross out incorrect work.

Good luck! ☺

Question:	1	2	3	4	Total
Points:	20	25	25	30	100
Score:					

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1. (a) (15 points) Find all the solutions of the differential equation

$$\frac{dy}{dx} = 3x^2y + y.$$

$$= (3x^2+1)y$$

so $\displaystyle\int \frac{dy}{y} = \int (3x^2+1)\,dx$ or $\underbrace{y=0}$

$\ln|y| = x^3 + x + C$

$|y| = e^{x^3+x+C}$

$y = Ae^{x^3+x}$, A arbitrary

(b) (5 points) Solve the initial value problem with $y(0) = 2$.

$2 = Ae^0 = A$

$y = 2e^{x^3+x}$

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2. (a) (10 points) Show that the following equation is exact:

$$\underbrace{3x^2 + 5e^{6y}}_{M} + (\underbrace{30xe^{6y} - 2y}_{N})y' = 0.$$

Check $M_y = N_x$:

$M_y = 30e^{6y}$

$N_x = 30e^{6y}$ ✓

Yes, it is exact ✓

(b) (15 points) Find the general solution of this differential equation. *It suffices to give the solution implicitly.*

Want $\psi(x,y)$ so that $\psi_x = M$, $\psi_y = N$

$\int M\,dx = x^3 + 5xe^{6y}$

$\int N\,dy = 5xe^{6y} - y^2$

$\Rightarrow \psi(x,y) = x^3 - y^2 + 5xe^{6y}$

Sol'n: $\boxed{x^3 - y^2 + 5xe^{6y} = C}$

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3. (25 points) Solve the initial value problem

$$y'' + 2y' - 3y = 0, \qquad y(-1) = e^3 - e^{-1}, \quad y'(-1) = -3e^3 - e^{-1}.$$

$r^2 + 2r - 3 = 0 \Rightarrow r_{1,2} = \dfrac{-2 \pm \sqrt{4+12}}{2} \Rightarrow$

$\Rightarrow r_1 = -3, \ r_2 = 1$

$\Rightarrow y(t) = c_1 e^{-3t} + c_2 e^t \Rightarrow y'(t) = -3c_1 e^{-3t} + c_2 e^t$

$y(-1) = e^3 - e^{-1} \Rightarrow c_1 e^3 + c_2 e^{-1} = e^3 - e^{-1}$

$y'(-1) = -3e^3 - e^{-1} \Rightarrow -3c_1 e^3 + c_2 e^{-1} = -3e^3 - e^{-1}$ $(-)$

$\Rightarrow 4c_1 e^3 = 4e^3 \Rightarrow c_1 = 1 \Rightarrow c_2 = -1.$

$\Rightarrow y(t) = e^{-3t} - e^t$

4. An African government is trying to come up with a good policy regarding the hunting of hippopotami. They are using the following model: the hippo population has a natural growth rate of 10% a year and suppose a constant hunting rate of a hippos per year.

   (a) (10 points) Write down a differential equation that models the hippo population. Justify your answer. *: Your first step should be to fix symbols and units.* Use $x(t)$ to denote the number of hippos at time t.

$$\underbrace{\frac{dx}{dt}}_{\text{hippos/year}} = \underbrace{(\text{rate of birth} - \text{natural death})}_{=10\%}\cdot x(t) - \underbrace{a}_{\text{hunting}}.$$

hippos/year   10% hippos/year   hunting hippos/year

$$\frac{dx}{dt} = \frac{10}{100}x - a$$

$$\frac{dx}{dt} = \frac{x}{10} - a$$

   (b) (5 points) Suppose no hunting occurs. What is the time it takes the initial population to double? There might be some logs in the answer.

no hunting : $a=0$

$$\frac{dx}{dt} = \frac{x}{10}$$

$$x = ce^{\frac{t}{10}}$$

$x(0) = c$

$x(t) = 2c \Rightarrow ce^{\frac{t}{10}} = 2c \Rightarrow e^{\frac{t}{10}} = 2$

$t = 10\ln 2$

(c) (10 points) Find the general solution of this equation when $a = 3$.

equation: $\frac{dx}{dt} = \frac{x}{10} - 3$

$$\frac{dx}{dt} = \frac{x-30}{10}$$

$$\frac{dx}{x-30} = \frac{1}{10}dt$$

$$\int \frac{dx}{x-30} = \frac{1}{10}dt$$

$$\ln|x-30| = \frac{1}{10}t + c_1$$

$$x - 30 = ce^{\frac{1}{10}t} \Rightarrow x = 30 + ce^{\frac{t}{10}}$$

(d) (5 points) What is the equilibrium solution when $a = 3$? Is it a stable, unstable or semistable equilibrium? Explain why.

equilibrium: $\frac{x}{10} - 3 = 0$

$$x = 30$$

phase line

$\frac{dx}{dt} < 0$ $\frac{dx}{dt} > 0$

$\longleftarrow \; 30 \; \longrightarrow$

This is an unstable equilibrium.