MATH 20D

Exam 1 v5

Lecture A

April 21, 2015

Name: SOLUTIONS

PID: ____________________

TA: ____________________

Section #: ________ Section time: ____________________

Seat: ____________________

There are 6 pages and 4 questions, for a total of 100 points.

No calculators, no electronic devices, no books, no notes, except for the one 8.5in×11in sheet of notes.

Please turn off all electronic devices.

Answer the questions in the spaces provided on the question sheets. Unless otherwise stated, show all your work for full credit. To maximize credit, cross out incorrect work.

Good luck! ☺

Question:	1	2	3	4	Total
Points:	20	25	25	30	100
Score:					

Name: ______________________
Section #: Section time:

1. (a) (15 points) Find all the solutions of the differential equation

$$\frac{dy}{dx} = \frac{x(y-1)+y-1}{x}.$$

$$= \frac{(x+1)(y-1)}{x}$$

so $\displaystyle\int \frac{dy}{y-1} = \int \frac{x+1}{x}\,dx = \int 1 + \frac{1}{x}\,dx$ or $y=1$

$$\ln|y-1| = x + \ln|x| + C$$

$$|y-1| = e^{x+\ln|x|+C}$$

$$= Ae^{x}e^{\ln|x|}, \quad A>0$$

$$y-1 = Ae^{x}x, \quad A \neq 0$$

$$y = Axe^{x}+1, \quad A \text{ arbitrary}$$

(b) (5 points) Solve the initial value problem with $y(1) = 2e+1$.

$$2e+1 = A(1)e^{1}+1$$

$$= Ae+1$$

so $A = 2$

$$y = 2xe^{x}+1$$

Name:

Section #: Section time:

2. (a) (10 points) Show that the following equation is exact:

$$\underbrace{3y - 12ye^{-3x}}_{M} + \underbrace{(4e^{-3x} + 3x + 2y)}_{N}y' = 0.$$

Check $M_y = N_x$:

$M_y = 3 - 12e^{-3x}$

$N_x = -12e^{-3x} + 3$

Yes, it is exact ✓

(b) (15 points) Find the general solution of this differential equation. *It suffices to give the solution implicitly.*

Want to find $\psi(x,y)$ s.t. $\psi_x = M$ and $\psi_y = N$

$\int M\,dx = 3xy + 4ye^{-3x}$

$\int N\,dy = 4ye^{-3x} + 3xy + y^2$

So $\psi(x,y) = 4ye^{-3x} + 3xy + y^2$

Sol'n: $\boxed{4ye^{-3x} + 3xy + y^2 = C}$

3. (25 points) Solve the initial value problem

$$y'' + 9y = 0, \qquad y(\pi) = -2, \quad y'(\pi) = 2.$$

$r^2 + 9 = 0 \Rightarrow r_{1,2} = \pm 3i$

$\Rightarrow y(t) = c_1 \cos 3t + c_2 \sin 3t$

$\Rightarrow y'(t) = -3c_1 \sin 3t + 3c_2 \cos 3t$

$y(\pi) = -2 \Rightarrow -c_1 = -2 \Rightarrow c_1 = 2$

$y'(\pi) = 2 \Rightarrow -3c_2 = 2 \Rightarrow c_2 = \frac{2}{3}$

$\Rightarrow y(t) = 2\cos 3t + \frac{2}{3} \sin 3t$

4. An Asian government is trying to come up with a good policy regarding the hunting of tigers. They are using the following model: the tiger population has a natural growth rate of 5% a year and suppose a constant hunting rate of a tigers per year.

   (a) (10 points) Write down a differential equation that models the tiger population. Justify your answer. : *Your first step should be to fix symbols and units.* Use $x(t)$ to denote the number of tigers at time t.

$x(t)$ = # tigers at time t

$$\frac{dx}{dt} = \underbrace{(\text{birth} - \text{death})\text{ ratio}}_{\frac{5}{100}\,/\text{year}} \cdot x - \underbrace{\text{hunting}}_{\text{constant} = a}$$

$$\underset{\text{tigers/year}}{\frac{dx}{dt}} = \underset{1/\text{year}}{\frac{5}{100}}\ \underset{\text{tiger}}{x} - \underset{\text{tigers/year}}{a} \Rightarrow \boxed{\frac{dx}{dt} = \frac{x}{20} - a}$$

   (b) (10 points) Solve the equation.

$$\frac{dx}{dt} = \frac{x}{20} - a$$

$$\frac{dx}{dt} = \frac{x - 20a}{20}$$

$$\frac{dx}{x - 20a} = \frac{dt}{20} \Rightarrow x - 20a = ce^{\frac{t}{20}}$$

$$x = 20a + ce^{\frac{t}{20}}$$

(c) (10 points) Before instituting their policy, they take a census of the tigers and find out that there are 160 individuals left. Their goal is to reach a population of 400 tigers in $t_1 = 20 \ln 3$ years. What should the hunting rate a be to reach this goal?

$x(0) = 160$

$x = 20a + Ce^{\frac{t}{20}}$ $\Rightarrow$ $20a + C = 160$

$x(20\ln 3) = 400 \Rightarrow 20a + Ce^{\frac{20\ln 3}{20}} = 400$

$20a + Ce^{\ln 3} = 400$

$20a + 3C = 400$

$20a + C = 160$
$20a + 3C = 400$ $\Rightarrow 2C = 400 - 160 = 240$

$C = 120$

$20a = 40$

$\boxed{a = 2}$ tigers/year