Some Maclaurin Series and Their Intervals of Convergence

$$e^x = 1 + \tfrac{1}{1!}x + \tfrac{1}{2!}x^2 + \tfrac{1}{3!}x^3 + \cdots = \sum_{n=0}^{\infty} \frac{1}{n!} \cdot x^n \qquad (-\infty, \infty)$$

$$\sin x = x - \tfrac{1}{3!}x^3 + \tfrac{1}{5!}x^5 - \tfrac{1}{7!}x^7 + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \cdot x^{2n+1} \qquad (-\infty, \infty)$$

$$\cos x = 1 - \tfrac{1}{2!}x^2 + \tfrac{1}{4!}x^4 - \tfrac{1}{6!}x^6 + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot x^{2n} \qquad (-\infty, \infty)$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots = \sum_{n=0}^{\infty} x^n \qquad (-1, 1)$$

$$\ln(1-x) = x + \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 + \tfrac{1}{4}x^4 + \cdots = \sum_{n=1}^{\infty} \frac{1}{n} \cdot x^n \qquad [-1, 1)$$

$$\arctan x = x - \tfrac{1}{3}x^3 + \tfrac{1}{5}x^5 - \tfrac{1}{7}x^7 + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \cdot x^{2n+1} \qquad [-1, 1]$$

We can use substitution technique, derivation or integration term by term to easily obtain Maclaurin series for many other functions, e.g. $\frac{1}{1-x^2}$, e^{-2x}, etc.