HOMOGENEOUS SECOND ORDER ODE'S

$$ay'' + by' + cy = 0, \text{ where a,b,c, are real numbers with } a \neq 0$$

(1) Write down the associated quadratic equation $ar^2 + br + c = 0$.

(2) Compute its discriminant $\Delta = b^2 - 4ac$.

(3) There are three possible situations:

- If $\Delta > 0$, the quadratic equation has two distinct real solutions, r_1 and r_2. Find them. (You might need to use the quadratic formula.) The general solution of the differential equation is

$$y = C_1 e^{r_1 t} + C_2 e^{r_2 t}.$$

- If $\Delta = 0$, the quadratic equation has only one real root r. Find it. The general solution of the differential equation is

$$y = C_1 e^{rt} + C_2 t e^{rt}.$$

- If $\Delta < 0$, the quadratic equation does not have any real roots. Compute $\alpha = -\frac{b}{2a}$ and $\beta = \frac{\sqrt{|\Delta|}}{2a}$. The general solution of the differential equation is

$$y = C_1 e^{\alpha t} \cos(\beta t) + C_2 e^{\alpha t} \sin(\beta t).$$

(4) If it is an initial value problem or a boundary problem, plug in the given values and solve for C_1 and C_2. Don't forget to take the derivative of y in the case of an initial value problem (chain rule!).