1. Check that both $\cos(\omega t)$ and $\sin(\omega t)$ satisfy the *harmonic oscillator* equation

$$x'' + \omega^2 x = 0$$

2. Among the functions $x(t) = a\cos(\omega t) + b\sin(\omega t)$, which ones have $x(0) = 0$? Doesn't this contradict the uniqueness theorem for differential equations?

3. Suppose r is a (perhaps complex) constant such that e^{rt} is a solution to $x'' + \omega^2 x = 0$. What does r have to be?

4. More fun with complex numbers. Say $r = a + ib$ and $w = Ae^{-i\phi t}$ are complex numbers. What is the real part of we^{rt}? Can you choose w so that $we^{rt} = Im(e^{rt})$?

5. Now look at the handout given in class. Solve the following equations and draw the solutions with $y(0) = 0, y'(0) = 1$ and $y(0) = 0, y'(0) = 1$.

   (a) $y'' + 3y' - 10y = 0$

(b) $y'' + 8y' + 25y = 0$

(c) $4y'' - 12y' + 9y = 0$

6. Suppose $e^{-t/2}\cos(3t)$ is a solution of the equation $mx'' + bx' + kx = 0$.

   (a) What can you say about m, b, k?

   (b) What is an exponential solution to this equation?

   (c) Sketch the curve in the complex plane traced by one of the exponential solutions. Then sketch the graph of the real part and explain the relationship.

7. For what value of b does the equation

$$x'' + bx' + x = 0$$

   (a) exhibit critical damping?
   (b) have oscillatory solutions?
   (c) have *only* damped solutions?
   (d) Describe how the pseudo-period of the solution changes as b increases from 0.

8. Find a second solution y_2 to $y'' - 2y' + y = 0$, given that one solution is $y_1 = e^x$ by setting $y_2(x) = u(x)e^x$ and determining $u(x)$ by substituting into the ODE.

9. A series RLC-circuit is modeled by either of the ODEs

$$LQ'' + RQ' + \frac{Q}{C} = E(t)$$

$$LI'' + RI' + \frac{I}{C} = E'(t)$$

where $Q(t)$ is the charge on the capacitor, $I(t)$ is the current in the circuit, $E(t)$ is the applied electromotive force (from a battery or a generator), and the constants L, R, C are respectively the inductance of the coil, the resistance, and capacitance, measured in some compatible system of units. Recall that $Q' = I$.

(a) What is the relation between the two equations?

(b) Show that if $R = 0$ and $E = 0$, then $Q(t)$ varies periodically, and find the period, assuming that $L \neq 0$.

(c) Assume $E = 0$. How must R, L, C be related if the current oscillates?

10. Show that the angle α of the pendulum swinging with small amplitude (so you can approximate $\sin\alpha \approx \alpha$) approximately obeys a second-order ODE with constant coefficients. Use

$$L = \text{length}, \quad m = \text{mass}, \quad \text{damping} = mc\frac{d\alpha}{dt} \text{ for some constant } c.$$

If the motion is undamped (that is, $c = 0$), express the period in terms of L, m, and the gravitational constant g.