# Step and delta functions; step and delta responses

1. Suppose $q(t) = 2u(t-1) + \delta(t-2) - 2u(t-3)$. Sketch a graph of this generalized function. Formulate at least one scenario which might result in each of the equations

   $x' + kx = q(t)$ (your choice of $k$, it might be negative);

   $2x'' + 4x' + 4x = q(t)$.

2. (a) Graph the functions

   $$f(t) = 3\,(u(t-1) - u(t-2))\,t$$

   and

   $$g(t) = \begin{cases} 0 & t < 0 \\ \lfloor t \rfloor & t > 0 \end{cases}$$

   where $\lfloor t \rfloor$ denotes the greatest integer less than or equal to $t$.

   (b) Express $f(t)$ as an alternative ($f(t) = \cdots$ for $t < \cdots$, etc). Express $g(t)$ as a single formula using the step function $u(t)$.

   (c) Using the graphs of $f(t)$ and $g(t)$, graph the generalized derivatives of these two functions. Use labeled harpoons to denote the delta functions that occur.

   (d) Finally, differentiate formally the expression for $f(t)$ that was given and the expression for $g(t)$ you found in part (b). Graph the resulting functions.

3. Find the unit step function and unit impulse responses to the operator $D^2 + 2D + 2I$, and graph them. Why is one the derivative of the other? How do these results change if one uses instead the operator $2D^2 + 4D + 4I$?

4. Using the time-invariance and your solution to part 3, write down a solution to

   $$x'' + 2x' + 2x = 2u(t-1) + \delta(t-2) - 2u(t-3).$$