

**HOMEWORK 7 – PART I**

DUE 11 NOVEMBER 2008

This part deals with quadratic forms. Reading: section 6 in Davenport's book.

1. Prove that any quadratic form with integer coefficients is equivalent to a quadratic form  $Q = ax^2 + bxy + cy^2$  with  $|b| \leq |a| \leq |c|$ .
2. Prove that there is only one equivalence class of positive definite forms of discriminant  $-4$ .
3. Given integers  $m, n$  such that  $m^2 - dn^2 = 4$ , find the automorph of quadratic forms of discriminant  $d$  induced by  $m$  and  $n$ .
4. If  $d > 0$ , prove that the integral solutions to Pell's equation  $x^2 - dy^2 = 4$  are

$$\left\{ (t, u); \frac{t + u\sqrt{d}}{2} = \pm \left( \frac{m + n\sqrt{d}}{2} \right)^k, \quad k \in \mathbb{Z} \right\},$$

where  $(m, n)$  is the positive integral solution with  $n$  minimal.