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Outline of (main) projects for 2013-2016:

Semiclassical Isospectral Problems.
One of my primary interests is to use microlocal analysis and symplectic geometry to solve problems in spectral theory. I work primarily on isospectrality problems concerning semiclassical quantum systems. These problems go back to the works of Colin de Verdière, Duistermaat, Guillemin, and Sjöstrand in the 1970s/1980s.

Two years ago I began to study major contributions to the isospectral problem for Riemannian manifolds that Sarnak had made with Osgood and Phillips. They showed that, in dimension two, the isospectral set (while not finite) is compact in C^∞. I had considered analogue problems for collections of commuting operators. Jointly with Charles and Vũ Ngọc I proved that *quantum toric integrable systems*, which are given by r independendent commuting operators with principal symbols that generate periodic flows, are determined by their semiclassical joint spectrum. This solves the isospectral problem in full for quantum toric systems (published in a paper dedicated to P. Sarnak). In view of this work, Sarnak asked me in 2011 a question concerning r independent commuting operators: *how much can one improve the* L^∞ *bounds for the joint eigenfunctions he obtained?* Sarnak had shown that in the case of the commutative ring of differential operators of rank r, one gets an optimal improvement depending on r. I am working on this now.

My other goal in the framework of isospectrality questions is proving that the joint semiclassical spectrum of a *quantum semitoric integrable system* (given by operators whose principal symbols are all periodic with the exception of possibly one) determines the classical system given by the principal symbols of the operators. This would solve the isospectrality problem for *quantum semitoric systems*. Semitoric systems retain some of the rigidity properties of toric systems, but can in addition have nodal singularities (corresponding to fibers which are multi-pinched tori). An example of such a system is the Jaynes-Cummings model from optics.

Semiclassical Symplectic Capacities.
A major result in symplectic topology is Gromov's Nonsqueezing theorem (1985), which shows the existence of symplectic capacities. Symplectic capacities were introduced by Ekeland and Hofer in 1989, who came from the angle of Hamiltonian dynamics. The goal of this project, which is joint with Charles, Polterovich, and Vũ Ngọc, is investigating a semiclassical notion of symplectic capacity. We have some understanding of preliminary examples, following recent insights of Polterovich.

Stochastic Arnol'd Conjecture.
I am interested in probability and in applying it to problems in symplectic geometry. This is a project with Rezakhanlou. Arnol'd formulated the higher dimensional analogue of the Poincaré-Birkhoff Theorem: the Arnol'd Conjecture (1978). The first breakthrough on the conjecture was by Conley and Zehnder (1983) who proved it for the $2n$-torus. A proof for the $2n$-torus using generating functions was later given by Chaperon (1984). The goal is proving a random Arnol'd Conjecture, as continuation of the work Rezakhanlou and I did on the random Poincaré-Birkhoff theorem, using random generating functions.