Math 20F Practice Midterm

11 July, 2008

1. True or False?

   (a) If the vectors $\vec{a_1}, \ldots, \vec{a_n}$ formed from the columns of a square matrix A satisfy $2\vec{a_1} + \pi\vec{a_2} = 17\vec{a_3}$ then $A\vec{x} = \vec{0}$ has infinitely many solutions.

   (b) If A and B are square matrices with A singular and B nonsingular, then AB is nonsingular.

   (c) If $\vec{x_1}, \vec{x_2}, \ldots, \vec{x_n}$ are vectors in $\mathbb{R}^n$ such that $\vec{x_1}$ is in $\text{Span}\{\vec{x_2}, \vec{x_3}, \ldots, \vec{x_n}\}$ then $\text{Span}\{\vec{x_1}, \vec{x_2}, \ldots, \vec{x_n}\}$ is not all of $\mathbb{R}^n$.

   (d) If $\vec{x_1}, \vec{x_2}, \ldots, \vec{x_n}$ are vectors in $\mathbb{R}^n$ such that $\text{Span}\{\vec{x_1}, \vec{x_2}, \ldots, \vec{x_n}\}$ is not all of $\mathbb{R}^n$ then $\{\vec{x_1}, \vec{x_2}, \ldots, \vec{x_n}\}$ is linearly dependent.

   (e) If $A\vec{p} = \vec{b}$ and $A\vec{v} = \vec{0}$ then $A(\vec{p} + 3\vec{v}) = \vec{0}$.

   (f) If T is the linear transformation mapping $\mathbb{R}^n$ to $\mathbb{R}^n$ given by $T(\vec{x}) = A\vec{x}$ and $\det(A) \neq 0$ then for any $\vec{b}$ in $\mathbb{R}^n$ there is an $\vec{x}$ so that $T(\vec{x}) = \vec{b}$.

   (g) If A has nonzero determinant then A^{-1} does as well.

   (h) Let $\vec{x_1}, \vec{x_2}, \ldots, \vec{x_n}$ be vectors in $\mathbb{R}^n$ and suppose that $c_1\vec{x_1} + c_2\vec{x_2} + \cdots + c_n\vec{x_n} = \vec{0}$. It follows that $c_1 = c_2 = \cdots = c_n = 0$.

   (i) If $\vec{u}$ and $\vec{v}$ are two vectors in a vector space V then $4\vec{u} + 101\vec{v}$ is also in V.

   (j) If A is an 4×4 matrix with nonzero determinant then, for any $\vec{b}$ in $\mathbb{R}^4$, there is an $\vec{x}$ so that $A\vec{x} = \vec{b}$.

   (k) If the matrix equation $A\vec{x} = \vec{b}$ has a solution then $\vec{b}$ is in the column space of A.

2. Let $A = \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & a \\ 2 & b & -6 \\ -3 & -6 & -3 \end{bmatrix}$, $\vec{u} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

(a) Compute $B(\vec{u} - 3\vec{v})$.

(b) Compute $\det(A)$.

(c) Is A singular?

(d) Find a and b so that B is the cofactor matrix of A (the matrix whose $i - j^{th}$ entry is the $i - j^{th}$ cofactor of A)and find A^{-1} using Cramer's rule. (You can check your answer by computing AA^{-1}.)

(e) Find $\vec{x}$ so that $A\vec{x} = \vec{u}$. (You can check your answer by computing $A\vec{x}$.)

3. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $\vec{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} a \\ b \end{bmatrix}$.

(a) Compute $\det(A)$.

(b) Find A^{-1}.

(c) Find $\vec{x}$ so that $A\vec{x} = \vec{u}$.

(d) Find $\vec{x}$ so that $A\vec{x} = \vec{v}$.

(e) What is $\text{Span}\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\}$?

(f) Find c_1 and c_2 such that $\vec{u} = c_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 4 \end{bmatrix}$.

(g) Is the set of vectors formed by the columns of A a linearly dependent set? Why or why not?

4. Assume the vectors formed by the columns of a square matrix A span $\mathbb{R}^n$.

   (a) Show that $A\vec{x} = \vec{b}$ has a solution for every $\vec{b}$ in $\mathbb{R}^n$ without appealing to the Invertible Matrix Theorem.

   (b) Show that the set of vectors formed from the columns of A is an independent set without appealing to the Invertible Matrix Theorem.