Homework due Thursday, October 12, at 3:00 pm.

A. Let $F$ be the set of all rational functions

$$
\frac{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0}{b_m x^m + b_{m-1} x^{m-1} + \cdots + b_0}
$$

(1)

where the coefficients are real numbers and $b_m \neq 0$.

(i) Define addition and multiplication of two elements in $F$ to be the usual addition and multiplication of functions. Show that with this addition and multiplication, $F$ is a field.

(ii) We can define an order on $F$ as follows. A rational function like (1) is positive if and only if $a_n$ and $b_m$ have the same sign, i.e. $a_n b_m > 0$. Now given two rational functions $\frac{p}{q}$ and $\frac{f}{g}$ we define:

$$
\frac{p}{q} > \frac{f}{g} \text{ if and only if } \frac{p}{q} - \frac{f}{g} > 0.
$$

Show with this ordering and the operations in part (i), $F$ is an ordered field.

(iii) Write the following polynomials in order of increasing size using the order define in (ii): $x^2$, $-x^5$, 2, $x + 6$, $3 - 2x$.

(iv) Show that $x > a$ for all $a \in \mathbb{R}$.

B. Let $S$ and $T$ be two bounded, nonempty, subsets of the set of positive real numbers. Define $ST := \{st : s \in S, t \in T\}$. Prove that

$$
\sup(ST) = \sup S \sup T.
$$

C. Rudin, Chapter 1 (page 21), problems #7, 8, 13.