Homework due Friday, October 20, at 3:00 pm.

A. Recall the definition of absolute value. For any real number \( x \) we define

\[
|x| = \begin{cases} 
  x & \text{if } x \geq 0 \\
  -x & \text{if } x < 0
\end{cases}
\]

(a) Prove that for all real numbers \( x \) and \( y \) we have \(|x + y| \leq |x| + |y|\).

(b) Prove that for all real numbers \( x \) and \( y \) we have \(||x| - |y|| \leq |x - y|\).

(c) Prove that for all real numbers \( x \) and \( y \) we have \(|xy| = |x||y|\).

(d) Prove that for all real number \( x \) we have \(|x|^2 = |x^2| = x^2\).

(e) Prove that for all real number \( x \) we have \(\sqrt{|x^2|} = |x|\). (Recall that by the definition \( z = \sqrt{y} \) if and only if \( z \geq 0 \) and \( z^2 = y \).

(f) Prove that for all real numbers \( x \) and \( y \) we have \(x^2 \leq y^2 \iff |x| \leq |y|\).

B. Let \( d \) and \( n \) be positive integers. Assume that \( d|n \) and \( d|n + 1 \). Prove that \( d = 1 \).

C. Eccles, Problems I which begin on page 53, problems # 12, 13, 14, 16.