Homework due Thursday, October 19, at 3:00 pm.

A. Theorem 1.11 (You do not need to include a written version of this proof in your HW. But you should prove the statement and/or read the proof in Rudin and make sure it makes sense).

B. For any positive integer \( n \), define
\[
\mathbb{C}^n = \{(z_1, \ldots, z_n) : z_j \in \mathbb{C}\}.
\]
For any two elements \( z = (z_1, \ldots, z_n) \) and \( w = (w_1, \ldots, w_n) \) in \( \mathbb{C}^n \), define
\[
z + w = (z_1 + w_1, \ldots, z_n + w_n).
\]
For any \( z = (z_1, \ldots, z_n) \) in \( \mathbb{C}^n \) and any \( \lambda \in \mathbb{C} \), define
\[
\lambda z = (\lambda z_1, \ldots, \lambda z_n).
\]
Define a scalar product on \( \mathbb{C}^n \) as follows. Let \( z = (z_1, \ldots, z_n) \) and \( w = (w_1, \ldots, w_n) \) be in \( \mathbb{C}^n \), define
\[
(z, w) = \sum_{j=1}^{n} z_j w_j.
\]
Let \( z, w \in \mathbb{C}^n \) and let \( \lambda \in \mathbb{C} \).

(a) Show that \((\lambda z, w) = \lambda (z, w) = (z, \lambda w)\).

(b) Show that \((z, z)\) is a non-negative real number. Moreover, \((z, z) = 0\) if and only if \( z = (0, \ldots, 0) \).

(c) Show that \((z, w) = (w, z)\).

(d) Assume \( w \neq (0, \ldots, 0) \) and let \( u = z - \frac{(z, w)}{(w, w)} w \). Show that \((u, w) = 0\).

(e) Compute \((u, u)\). Then use part (b) to prove the Cauchy-Schwarz inequality.

C. Give an explicit one-to-one correspondence between

(a) the points of two open intervals,
(b) the points of two closed intervals,
(c) the points of a closed interval and the points of an open interval,
(d) the points of the closed interval \([0, 1]\) and the set \( \mathbb{R} \).

D. Rudin, Chapter 1 (page 21), problems # 14, 17.

E. Rudin, Chapter 2 (page 43), problems # 2, 3, 5, 6, 7, 8.