Show all your computations and justify/explain your answers. Electronic devices are NOT allowed.

(1) Let \( \sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 3 & 1 & 2 & 6 & 7 & 5 & 8 \end{pmatrix} \).

(a) (6 pts) Write, with justification, \( \sigma \) as a product of disjoint cycles.

(b) (4 pts) Determine, with justification, whether \( \sigma \) is even or odd.

(2) Let \( \sigma = (1, 2)(5, 1, 3)(4, 2, 6, 7) \) be a permutation in \( S_8 \).

(a) (5 pts) Write \( \sigma \) as a product of disjoint cycles.

(b) (5 pts) Compute, with justification, \( \sigma^{100} \).

(3) (5 pts) Let \( H \) be the subgroup of \( (\mathbb{Z}_{20}, +_{20}) \) which is generated by 4. Find all the left cosets of \( H \) in \( \mathbb{Z}_{20} \).

(4) (5 pts) Prove or disprove by an example: suppose \( (G : H) = 3 \), then every right coset is a left coset.

(5) (5 pts) Let \( G \) be a finite group of order \( n \) and let \( e \in G \) denote the identity element. Show that \( g^n = e \) for all \( g \in G \).