Section A: Homework due Thursday, April 9, 9:00 pm, on Gradescope.

(1) Let $R = \mathcal{P}(\mathbb{N}_n)$ be the power set (the set of subsets) of $\mathbb{N}_n = \{1, 2, \ldots, n\}$. Define addition and multiplication on $R$ by

$$A + B := A \Delta B \quad \text{and} \quad A \cdot B := A \cap B.$$  

(Recall that $A \Delta B = (A \setminus B) \cup (B \setminus A)$.

(a) Show that $(R, +, \cdot)$ is a commutative ring. (You may assume without a proof that the symmetric difference is an associative operation.)

(b) Find the identity element for addition and multiplication.

(2) Exercises 18 page 174: 2, 4, 10, 12, 22, 28, 37

Section B: Extra practice problems: Problems in section B are for your practice; please do not hand them in. However, it is extremely important that you feel comfortable with these problems as some of them may appear on the exam(s).

(1) Find all the solutions of $x^2 - 5x + 6 = 0$ in

(a) $\mathbb{Z}_7$.
(b) $\mathbb{Z}_8$.
(c) $\mathbb{Z}_4 \times \mathbb{Z}_5$.

(2) Exercises 18 page 174: 6, 11, 15, 19, 20, 21, 44, 50