Section A: Homework due Thursday, April 23, 9:00 pm, on Gradescope.

(1) Let \( R \) be a ring with 1, and let \( u \in R \) be a unit. Let \( n \) be a positive integer so that \( nu = 0 \). Show that
\[
na = 0 \quad \text{for all } a \in R.
\]

(2) Exercise 20 page 189: 2, 4, 6, 8, 14

Section B: Extra practice problems: Problems in section B are for your practice; please do not hand them in. However, it is extremely important that you feel comfortable with these problems as some of them may appear on the exam(s).

(1) Let \( R \) be a ring without a zero divisor, i.e., if \( ab = 0 \) for some \( a, b \in R \), then either \( a = 0 \) or \( b = 0 \). Assume there exists some nonzero element \( x \in R \) so that \( x^2 = x \). Show that \( x \) is the multiplicative identity. That is:
\[
xy = yx = y \quad \text{for every } y \in R.
\]

[Note that since \( R \) is not assumed to have 1, we cannot simply factor \( x^2 - x = x(x - 1) \).]

(Hint: Use \( x^2 = x \) to show that \( x(xy - y) = 0 \) and \((yx - y)x = 0 \) for every \( y \), conclude the assertion from this.)

(2) Exercise 20 page 189: 18, 22, 23,