Homework due Friday, February 22, 3:00 pm.

1. Let $A$ and $B$ be two nonempty sets. Let $f : A \rightarrow B$ be a bijection. Define
   $\phi : S_A \rightarrow S_B$ by $\phi(\sigma) = f \circ \sigma \circ f^{-1}$. Show that $\phi$ is an isomorphism between
   $S_A$ and $S_B$.

2. Let $n \in \mathbb{N}$ be a positive integer. Define
   
   \[ R_n = \begin{bmatrix} \cos \frac{2\pi}{n} & -\sin \frac{2\pi}{n} \\ \sin \frac{2\pi}{n} & \cos \frac{2\pi}{n} \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \]

   Also recall that $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ denotes the identity matrix. Let

   \[ G_n = \{ I, R_n, R_n^2, \ldots, R_n^{n-1}, X, R_nX, R_n^2X, \ldots, R_n^{n-1}X \}. \]

   (a) (Bonus problem) Show that $G_n$ is a group.
   (b) Let $n = 4$. Prove that $G_4$ is isomorphic to $D_4$ (the dihedral group of
       order 8). (Hint: Observe that $R_4$ is a rotation with angle $\pi/2$)

3. Exercise 8 page 83: 2, 8, 12, 21, 47, 49