Math Finance, Homework 3, Winter 2006
Due Wednesday, January 25, 2006

1. For the single period CRR model, in class we showed that

$$V_0(\phi^*) = \frac{1}{1+r}\left(\frac{1+r-d}{u-d}x^u + \frac{u-1-r}{u-d}x^d\right).$$

   Verify that the coefficients of $x^u/(1+r)$ and $x^d/(1+r)$ define a probability measure on the sample space Ω.

2. Let $(\Omega, \mathbf{P})$ be a probability space such that $\Omega = \{\omega_1, \omega_2\}$, $\mathbf{P}(\omega_1) = 1-p$, and $\mathbf{P}(\omega_2) = p$, where $0 < p < 1$. Let V be a nonnegative random variable defined on $(\Omega, \mathbf{P})$, i.e., $\mathbf{P}(V \geq 0) = 1$. If we regard V as a random variable defined on the probability space $(\Omega, \mathbf{P}^*)$, where $\mathbf{P}(\omega_1) = 1 - p^*$ and $\mathbf{P}(\omega_2) = p^*$ for some $0 < p^* < 1$, then $\mathbf{E}^*[V] \geq 0$. Show that

$$\mathbf{P}(V > 0) > 0$$

   if and only if

$$\mathbf{E}^*[V] > 0.$$

3. Consider a Single-period Binomial Model with $r = 1/3$, $S_0 = 2$, $d = 5/4$, $u = 3/2$, and $p = 3/5$. Let X be a European call option with strike price $K =$\$2.75 and expiration time $T = 1$. Compute the arbitrage free price.

4. Consider a Single-period Binomial Model with $r = 1/4$, $S_0 = 3$, $d = 1$, $u = 2$, and $p = 3/4$. Let X be a European put option with strike price $K =$\$5 and expiration time $T = 1$. Compute the arbitrage free price.

5. Problem 1 in Section 2.4 of the Williams notes.